PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM

Today’s Objectives:
Students will be able to:
1. Calculate the linear momentum of a particle and linear impulse of a force.
2. Apply the principle of linear impulse and momentum.

In-Class Activities:
• Check Homework
• Reading Quiz
• Applications
• Linear Momentum and Impulse
• Principle of Linear Impulse and Momentum
• Concept Quiz
• Group Problem Solving
• Attention Quiz
READING QUIZ

1. The linear impulse and momentum equation is obtained by integrating the ______ with respect to time.

   A) friction force  
   B) equation of motion  
   C) kinetic energy  
   D) potential energy

2. Which parameter is not involved in the linear impulse and momentum equation?

   A) Velocity  
   B) Displacement  
   C) Time  
   D) Force
A dent in an automotive fender can be removed using an impulse tool, which delivers a force over a very short time interval. To do so the weight is gripped and jerked upwards, striking the stop ring.

How can we determine the magnitude of the linear impulse applied to the fender?

Could you analyze a carpenter’s hammer striking a nail in the same fashion? Sure!
When a stake is struck by a sledgehammer, a large impulse force is delivered to the stake and drives it into the ground.

If we know the initial speed of the sledgehammer and the duration of impact, how can we determine the magnitude of the impulsive force delivered to the stake?
PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM  
(Section 15.1)

The next method we will consider for solving particle kinetics problems is obtained by integrating the equation of motion with respect to time.

The result is referred to as the principle of impulse and momentum. It can be applied to problems involving both linear and angular motion.

This principle is useful for solving problems that involve force, velocity, and time. It can also be used to analyze the mechanics of impact (taken up in a later section).
The principle of linear impulse and momentum is obtained by integrating the equation of motion with respect to time. The equation of motion can be written

\[ \sum F = m \ a = m \ (dv/dt) \]

Separating variables and integrating between the limits \( v = v_1 \) at \( t = t_1 \) and \( v = v_2 \) at \( t = t_2 \) results in

\[ \sum \int_{t_1}^{t_2} F \ dt = m \int_{v_1}^{v_2} dv = m v_2 - m v_1 \]

This equation represents the principle of linear impulse and momentum. It relates the particle’s final velocity \( (v_2) \) and initial velocity \( (v_1) \) and the forces acting on the particle as a function of time.
Linear momentum: The vector $m\mathbf{v}$ is called the linear momentum, denoted as $\mathbf{L}$. This vector has the same direction as $\mathbf{v}$. The linear momentum vector has units of (kg·m)/s or (slug·ft)/s.

Linear impulse: The integral $\int F \, dt$ is the linear impulse, denoted $I$. It is a vector quantity measuring the effect of a force during its time interval of action. $I$ acts in the same direction as $\mathbf{F}$ and has units of N·s or lb·s.

The impulse may be determined by direct integration. Graphically, it can be represented by the area under the force versus time curve. If $\mathbf{F}$ is constant, then

$$I = F (t_2 - t_1).$$
The two momentum diagrams indicate direction and magnitude of the particle’s initial and final momentum, $m\mathbf{v}_1$ and $m\mathbf{v}_2$. The impulse diagram is similar to a free body diagram, but includes the time duration of the forces acting on the particle.

The principle of linear impulse and momentum in vector form is written as

$$m\mathbf{v}_1 + \sum \int_{t_1}^{t_2} \mathbf{F} \, dt = m\mathbf{v}_2$$

The particle’s initial momentum plus the sum of all the impulses applied from $t_1$ to $t_2$ is equal to the particle’s final momentum.
IMPULSE AND MOMENTUM: SCALAR EQUATIONS

Since the principle of linear impulse and momentum is a vector equation, it can be resolved into its x, y, z component scalar equations:

\[
m(v_x)_1 + \sum_{t_1}^{t_2} \int F_x \, dt = m(v_x)_2
\]

\[
m(v_y)_1 + \sum_{t_1}^{t_2} \int F_y \, dt = m(v_y)_2
\]

\[
m(v_z)_1 + \sum_{t_1}^{t_2} \int F_z \, dt = m(v_z)_2
\]

The scalar equations provide a convenient means for applying the principle of linear impulse and momentum once the velocity and force vectors have been resolved into x, y, z components.
PROBLEM SOLVING

• Establish the x, y, z coordinate system.

• Draw the particle’s free body diagram and establish the direction of the particle’s initial and final velocities, drawing the impulse and momentum diagrams for the particle. Show the linear momenta and force impulse vectors.

• Resolve the force and velocity (or impulse and momentum) vectors into their x, y, z components, and apply the principle of linear impulse and momentum using its scalar form.

• Forces as functions of time must be integrated to obtain impulses. If a force is constant, its impulse is the product of the force’s magnitude and time interval over which it acts.
EXAMPLE

**Given:** A 0.5 kg ball strikes the rough ground and rebounds with the velocities shown. Neglect the ball’s weight during the time it impacts the ground.

**Find:** The magnitude of impulsive force exerted on the ball.

**Plan:**
1) Draw the momentum and impulse diagrams of the ball as it hits the surface.
2) Apply the principle of impulse and momentum to determine the impulsive force.
Solution:

1) The impulse and momentum diagrams can be drawn as:

\[ \int W \, dt \approx 0 \]
\[ \int F \, dt \]
\[ \int N \, dt \approx 0 \]

The impulse caused by the ball’s weight and the normal force \( N \) can be neglected because their magnitudes are very small as compared to the impulse from the ground.
2) The principle of impulse and momentum can be applied along the direction of motion:

\[ m\mathbf{v}_1 + \sum \int_{t_1}^{t_2} \mathbf{F} \, dt = m\mathbf{v}_2 \]

\[
\begin{align*}
0.5 (25 \cos 45^\circ \mathbf{i} - 25 \sin 45^\circ \mathbf{j}) + \int_{t_1}^{t_2} \mathbf{F} \, dt &= 0.5 (10 \cos 30^\circ \mathbf{i} + 10 \sin 30^\circ \mathbf{j}) \\
\Rightarrow &\quad \int_{t_1}^{t_2} \sum \mathbf{F} \, dt \\
&= 0.5 (10 \cos 30^\circ \mathbf{i} + 10 \sin 30^\circ \mathbf{j})
\end{align*}
\]

The impulsive force vector is

\[ \mathbf{I} = \int_{t_1}^{t_2} \sum \mathbf{F} \, dt = (4.509 \mathbf{i} + 11.34 \mathbf{j}) \text{ N} \cdot \text{s} \]

Magnitude: \[ I = \sqrt{4.509^2 + 11.34^2} = 12.2 \text{ N} \cdot \text{s} \]
1. Calculate the impulse due to the force.
   - A) 20 kg·m/s
   - B) 10 kg·m/s
   - C) 5 N·s
   - D) 15 N·s

2. A constant force $F$ is applied for 2 s to change the particle’s velocity from $v_1$ to $v_2$. Determine the force $F$ if the particle’s mass is 2 kg.
   - A) $(17.3 \, j) \, N$
   - B) $(-10 \, \hat{i} + 17.3 \, j) \, N$
   - C) $(20 \, \hat{i} + 17.3 \, j) \, N$
   - D) $(10 \, \hat{i} + 17.3 \, j) \, N$
GROUP PROBLEM SOLVING

Given: The 20 kg crate is resting on the floor. The motor M pulls on the cable with a force of $F$, which has a magnitude that varies as shown on the graph.

Find: The speed of the crate when $t = 6$ s.

Plan:
1) Determine the force needed to begin lifting the crate, and then the time needed for the motor to generate this force.
2) After the crate starts moving, apply the principle of impulse and momentum to determine the speed of the crate at $t = 6$ s.
Solution:

1) The crate begins moving when the cable force $F$ exceeds the crate weight. Solve for the force, then the time.

   $F = mg = (20) (9.81) = 196.2 \text{ N}$
   $F = 196.2 \text{ N} = 50 t$
   $t = 3.924 \text{ s}$

2) Apply the principle of impulse and momentum from the time the crate starts lifting at $t_1 = 3.924 \text{ s}$ to $t_2 = 6 \text{ s}$. Note that there are two external forces (cable force and weight) we need to consider.

   A. The impulse due to cable force:

   $$\int_{3.924}^{6} F \, dt = [0.5(250) 5 + (250) 1] - 0.5(196.2)3.924 = 490.1 \text{ N} \cdot \text{s}$$
B. The impulse due to weight:

$$+\uparrow \int_{3.924}^{6} (-mg) \, dt = -196.2 (6 - 3.924) = -407.3 \text{ N} \cdot \text{s}$$

Now, apply the principle of impulse and momentum

$$+\uparrow \quad mv_1 + \sum \int_{t_1}^{t_2} F \, dt = mv_2 \quad \text{where } v_1 = 0$$

$$0 + 490.1 - 407.3 = (20) \, v_2$$

$$\Rightarrow \quad v_2 = 4.14 \text{ m/s}$$
ATTENTION QUIZ

1. Jet engines on the 100 Mg VTOL aircraft exert a constant vertical force of 981 kN as it hovers. Determine the net impulse on the aircraft over $t = 10$ s.
   - A) $-981$ kN·s  
   - B) 0 kN·s  
   - C) 981 kN·s  
   - D) 9810 kN·s

2. A 100 lb cabinet is placed on a smooth surface. If a force of a 100 lb is applied for 2 s, determine the net impulse on the cabinet during this time interval.
   - A) 0 lb·s  
   - B) 100 lb·s  
   - C) 200 lb·s  
   - D) 300 lb·s
Example 15.1

The 100-kg stone is originally at rest on the smooth horizontally surface. If a towing force of 200 N, acting at an angle of 45°, is applied to the stone for 10 s, determine the final velocity and the normal force which the surface exerts on the stone during the time interval.
Example 15.1

Solution

Free-Body Diagram

Since all forces acting are constant, the impulses are

\[ I = F_c (t_2 - t_1) \]

Principle of Impulse and Momentum

Resolving the vectors along the x, y, z axes,

\[ \left( \begin{array}{c} + \\ \rightarrow \end{array} \right) \quad m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x \, dt = m(v_x)_2 \]

\[ 0 + 200(10) \cos 45^\circ = (100)v_2 \quad \Rightarrow \quad v_2 = 14.1 \text{m/s} \]

\[ \left( + \uparrow \right) \quad m(v_y)_1 + \sum \int_{t_1}^{t_2} F_y \, dt = m(v_y)_2 \]

\[ 0 + N_C (10) - 981(10) + 200(10) \sin 45^\circ = 0 \quad \Rightarrow \quad N_C = 840 \text{N} \]
Example 15.3

Block A and B have a mass of 3 kg and 5 kg respectively. If the system is released from rest, determine the velocity of block B in 6 s.
Example 15.3

Solution

**Free-Body Diagram**
Since weight of each block is constant, the cord tensions will also be constant.

Since mass of pulley $D$ is neglected, the cord tension is $T_A = 2T_B$. 
Example 15.3

Solution

Principle of Impulse and Momentum

Block $A$:

\[
(\uparrow \downarrow) \quad m(v_A)_1 + \sum \int_{t_1}^{t_2} F_y \, dt = m(v_A)_2
\]

\[
0 - 2T_B(6) + 3(9.81)(6) = (3)(v_A)_2 \quad (1)
\]

Block $B$:

\[
(\uparrow \downarrow) \quad m(v_B)_1 + \sum \int_{t_1}^{t_2} F_y \, dt = m(v_B)_2
\]

\[
0 + 5(9.81)(6) - T_B(6) = (5)(v_B)_2 \quad (2)
\]
Example 15.3

Solution

*Kinematics*

We have \(2s_A + s_B = l\)

Taking time derivative yields \(2v_A = -v_B\)

When B moves downward A moves upward.

Sub this result into Eq. 1 and solving Eqs. 1 and 2 yields \((v_B)_2 = 35.8 \text{ m/s}\) and \(T_B = 19.2 \text{ N}\)
End of the Lecture

Let Learning Continue