Estimation of thermal contact resistance and temperature distributions in the pad/disc tribosystem

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1. Introduction

The ceramic–metal frictional materials are now extensively used in brake systems for their high thermal stability and wear resistance [1]. In the process of braking, the ceramic patch is pressed to its counterbody (disc, brake drum, rim of the wheel, etc.). As a result of friction on the contact surface, the kinetic energy is transformed into heat. The elements of the brakes are heated and, hence, the operation conditions for the friction patches become less favorable: the friction coefficient decreases and their wear intensifies, which may lead to emergency situations. Thus, the correct calculation of temperature is one of the most important issues in the design of brakes [2]. In the past, there have been many investigations focusing on the determination of temperature in brake systems [3–5]. However, to the best of the authors’ knowledge, there are few researches describing the realistic situation in the brake systems and considering the temperature drop at the interface between the pad and the disc. The accurate determination of temperature distributions in the brake systems requires considering the imperfect conditions of thermal contact at the interface.

Over the past decades, inverse analysis has become a valuable alternative when the direct measurement of data is difficult or the measuring process is very expensive, for example, the detection of contact resistance, the determination of heat transfer coefficients, the estimation of unknown thermophysical properties of new materials, the prediction of damage in the structure fields, the detection of fouling-layer profiles on the inner wall of a piping system, the optimization of geometry, the prediction of crevice and pitting in furnace wall, the determination of heat flux at the outer surface of a vehicle re-entry, and so on.

Although there have been many investigations considering the effect of contact resistance in different applications [6–9], nevertheless, the determination of thermal contact resistance at the interface has never been an easy task. The main objective of the present study is to develop an inverse analysis to estimate the thermal contact resistance for the tribosystem in Ref. [5], which consists of a sliding strip (the pad) over the surface of a semi-infinite foundation (the disc). An analysis of this kind poses significant implications on the study of the problems associated with sliding contact such as in a brake system. In this study, we present the conjugate gradient method [10–13] and the discrepancy principle [14] to estimate the time-varying thermal contact resistance by using the simulated temperature measurements. Subsequently, the distributions of temperature in the strip and foundation can be determined as well. The conjugate gradient method with an adjoint equation, also called Alifanov’s iterative regularization method, belongs to a class of iterative regularization techniques, which means that the regularization procedure is performed during the iterative processes, thus the determination of optimal regularization conditions is not needed. No prior information is used in the functional form of the thermal contact resistance variation with time. On the other hand, the discrepancy principle is used to terminate the iteration process in the conjugate gradient method.
Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>Bi</td>
<td>Biot’s number ( (=dh/k_f) )</td>
</tr>
<tr>
<td>Bis</td>
<td>Biot’s number ( (=dh/k_s) )</td>
</tr>
<tr>
<td>d</td>
<td>thickness of the strip ( (m) )</td>
</tr>
<tr>
<td>f</td>
<td>frictional coefficient</td>
</tr>
<tr>
<td>h</td>
<td>thermal contact conductance ( (W,m^{-2},K^{-1}) )</td>
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<tr>
<td>h_c</td>
<td>convection heat transfer coefficient ( (W,m^{-2},K^{-1}) )</td>
</tr>
<tr>
<td>f_M</td>
<td>functional</td>
</tr>
<tr>
<td>f_p</td>
<td>gradient of functional</td>
</tr>
<tr>
<td>k</td>
<td>thermal conductivity ( (W,m^{-1},K^{-1}) )</td>
</tr>
<tr>
<td>P</td>
<td>compressive pressure ( (N,m^{-2}) )</td>
</tr>
<tr>
<td>p</td>
<td>direction of descent</td>
</tr>
<tr>
<td>q</td>
<td>intensity of the frictional heat flux ( (W,m^{-2}) )</td>
</tr>
<tr>
<td>T</td>
<td>temperature ( (K) )</td>
</tr>
<tr>
<td>T_0</td>
<td>temperature scaling factor ( (K) )</td>
</tr>
<tr>
<td>t</td>
<td>time coordinate ( (s) )</td>
</tr>
<tr>
<td>V</td>
<td>sliding velocity ( (m,s^{-1}) )</td>
</tr>
<tr>
<td>x</td>
<td>spatial coordinate ( (m) )</td>
</tr>
<tr>
<td>Y</td>
<td>measured temperature ( (K) )</td>
</tr>
<tr>
<td>y</td>
<td>spatial coordinate ( (m) )</td>
</tr>
<tr>
<td>z</td>
<td>spatial coordinate ( (m) )</td>
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Greek symbols

<table>
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<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>( \Delta )</td>
<td>small variation quality</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>thermal diffusivity ( (m^2,s^{-1}) )</td>
</tr>
<tr>
<td>( \beta )</td>
<td>step size</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>conjugate coefficient</td>
</tr>
<tr>
<td>( \eta )</td>
<td>very small value</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>variable used in the adjoint problem</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>standard deviation</td>
</tr>
<tr>
<td>( \tau )</td>
<td>transformed time coordinate</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>random variable</td>
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Superscripts/subscripts

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>K</td>
<td>iterative number</td>
</tr>
<tr>
<td>m</td>
<td>measurement position</td>
</tr>
<tr>
<td>*</td>
<td>dimensionless quantity</td>
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2. Analysis

2.1. Direct problem

To illustrate the methodology for developing expressions for the use in estimating the unknown time-dependent thermal contact resistance during frictional heating at the interface of a tribosystem, which consists of a sliding strip (the pad) over the surface of a semi-infinite foundation (the disc), we consider the following transient heat transfer problem of friction process as shown in Fig. 1. Here, the imperfect heat contact between the strip and the foundation is assumed. It is supposed that the constant compressive pressures \( P \) are applied to the upper surface of the strip and to the infinity of the foundation. The strip slides with uniform velocity \( V \) in the direction of \( y \)-axis on the semi-infinite surface. Considering the thermal contact resistance between the strip and the foundation, then the dimensionless governing equations and the associated boundary and initial conditions for the system can be written as [5]:

\[
\frac{\partial^2 T_f^\prime(z^\prime, t^\prime)}{\partial z^\prime^2} = \frac{\partial T_f^\prime(z^\prime, t^\prime)}{\partial t^\prime}, \quad 0 \leq z^\prime \leq 1, \quad t^\prime > 0.
\] (1)

Fig. 1. Geometry and coordinate system.

\[
\frac{\partial^2 T_f^\prime(z^\prime, t^\prime)}{\partial z^\prime^2} = \frac{1}{\alpha_f} \frac{\partial T_f^\prime(z^\prime, t^\prime)}{\partial t^\prime}, \quad -\infty < z^\prime < 0, \quad t^\prime > 0.
\] (2)

\[
\frac{\partial T_f^\prime(z^\prime, t^\prime)}{\partial z^\prime} + \text{Bis} \cdot T_f^\prime(z^\prime, t^\prime) = 0, \quad -1 < z^\prime < 0, \quad t^\prime > 0.
\] (3)

\[
T_f^\prime(z^\prime, t^\prime) \rightarrow 0, \quad z^\prime \rightarrow -\infty, \quad t^\prime > 0.
\] (4)

\[
k_f \frac{\partial T_f^\prime(0, t^\prime)}{\partial z^\prime} - \frac{\partial T_f^\prime(0, t^\prime)}{\partial z^\prime} = 1, \quad 0 \leq z^\prime < 1, \quad t^\prime > 0.
\] (5)

\[
k_f \frac{\partial T_f^\prime(0, t^\prime)}{\partial z^\prime} = \frac{\partial T_f^\prime(0, t^\prime)}{\partial z^\prime} = \text{Bis} \cdot \left[ T_f^\prime(0, t^\prime) - T_f^\prime(0, t^\prime) \right], \quad 0 \leq z^\prime < 1, \quad t^\prime > 0.
\] (6)

\[
T_f^\prime(z^\prime, 0) = 0, \quad 0 < z^\prime \leq 1, \quad t^\prime = 0.
\] (7)

\[
T_f^\prime(z^\prime, 0) = 0, \quad -\infty < z^\prime < 0, \quad t^\prime = 0.
\] (8)

where the subscripts \( s \) and \( f \) refer to the regions of strip and foundation, respectively.

The dimensionless variables used in the above formulation are defined as follows:

\[
z^\prime = z / d, \quad t^\prime = \alpha_t / \alpha_s, \quad T_0 = qd / k_s, \quad T_0^\prime = T_f / T_0,
\] (9)

\[
T_f^\prime = T_f / T_0, \quad \alpha_f = \alpha_f / \alpha_s, \quad k_f = k_f / k_s, \quad \text{Bi} = h_d / k_s.
\]

where \( d \) is the thickness of the strip, \( q = fVP \) is the frictional heat generation at the interface of the strip and the foundation. \( k \) and \( \alpha \) are the thermal conductivity and thermal diffusivity, respectively. The direct problem considered here is concerned with the determination of the medium temperature when the Biot numbers \( \text{Bis} \) and \( \text{Bis}(t^\prime) \), thermophysical properties of the system, and initial and boundary conditions are known. A hybrid numerical method of Laplace transformation and finite difference used in our previous work can be applied to solve the direct problem [13].

2.2. Inverse problem

For the inverse problem, the function of Biot number \( \text{Bis}(t^\prime) \) is regarded as being unknown, while everything else in Eqs. (1)–(8) is known. In addition, temperature readings taken at \( z = z_m \) in the foundation region are considered available. The objective of the inverse analysis is to predict the unknown time-dependent function...
of Biot number \(Bi(t')\), merely from the knowledge of these temperature readings. Let the measured temperature at the measurement position \(z = z_m\) and time \(t\) be denoted by \(Y(z_m,t)\). Then this inverse problem can be stated as follows: by utilizing the above mentioned measured temperature data \(Y(z_m,t)\), the unknown \(Bi(t')\) is to be estimated over the specified time domain.

The solution of the present inverse problem is to be obtained in such a way that the following functional is minimized:

\[
J[Bi(t')] = \int_{t'}^{\zeta} \left[ T_f(z_m,t') - Y'(z_m,t') \right]^2 dt',
\]

where \(Y'(z_m,t') = Y(z_m,t)/T_0\), and \(T_f(z_m,t')\) is the estimated (or computed) temperature at the measurement location \(z = z_m\). In this study, \(T_f(z_m,t')\) are determined from the solution of the direct problem given previously by using an estimated \(Bi(t')\) for the exact \(Bi(t')\), here \(Bi(t')\) denotes the estimated quantities at the \(k\)th iteration. \(\zeta\) is the final time of the measurement. In addition, in order to develop expressions for the determination of the unknown \(Bi(t')\), a 'sensitivity problem' and an 'adjoint problem' are constructed as described below.

2.3. Sensitivity problem and search step size

The sensitivity problem is obtained from the original direct problem defined by Eqs. (1)–(8) in the following manner: it is assumed that when \(Bi(t')\) undergoes a variation \(\Delta Bi(t')\), \(T_f(z',t')\) and \(T_f(z',t')\) are perturbed by \(T_i + \Delta T_i\) and \(T_f + \Delta T_f\), respectively. Then replacing in the direct problem \(Bi(t')\) by \(Bi(t') + \Delta Bi(t')\), \(T_i\) by \(T_i + \Delta T_i\), and \(T_f\) by \(T_f + \Delta T_f\), subtracting from the resulting expressions the direct problem, and neglecting the second-order terms, the following sensitivity problem for the sensitivity function \(\Delta T_i\) and \(\Delta T_f\) can be obtained:

\[
\frac{\partial^2 \Delta T_i}{\partial z'^2} = \frac{\partial \Delta T_i}{\partial t}, \quad 0 \leq z' \leq 1, \quad t' > 0,
\]

\[
\frac{\partial^2 \Delta T_f}{\partial z'^2} = \frac{1}{\alpha_p} \frac{\partial \Delta T_f}{\partial t'}, \quad -\infty < z' \leq 0, \quad t' > 0,
\]

\[
\frac{\partial \Delta T_i}{\partial z'} + Bi \cdot \Delta T_i = 0, \quad z' = 1, \quad t' > 0,
\]

\[
\Delta T_i \rightarrow 0, \quad z' \rightarrow -\infty, \quad t' > 0,
\]

\[
\frac{\partial \Delta T_i}{\partial z'} = 0, \quad z' = 0, \quad t' > 0,
\]

\[
\frac{\partial \Delta T_f}{\partial z'} + \frac{\partial \Delta T_f}{\partial t'} = Bi \left\{ \Delta T_i - \Delta T_f \right\}, \quad z' = 0, \quad t' > 0,
\]

\[
\Delta T_i = 0, \quad 0 \leq z' \leq 1, \quad t' = 0,
\]

\[
\Delta T_f = 0, \quad -\infty < z' \leq 0, \quad t' = 0.
\]

The sensitivity problem of Eqs. (11)–(18) can be solved by the same method as the direct problem of Eqs. (1)–(8).

2.4. Adjoint problem and gradient equation

To formulate the adjoint problem, Eqs. (1) and (2) are multiplied by the Lagrange multipliers (or adjoint functions) \(\lambda_i\) and \(\lambda_f\), respectively, and the resulting expressions are integrated over the time and correspondent space domains. Then the results are added to the right hand side of Eq. (10) to yield the following expression for the functional \(J[Bi(t')]\):

\[
J[Bi(t')] = \int_{\zeta}^{t} \left[ T_f(z_m,t') - Y'(z_m,t') \right]^2 \delta(z' - z_m) dz' dt' + \int_{\zeta}^{t} \left[ \frac{\partial T_f}{\partial z'} \delta(z' - z_m) \right] dz' dt' + \int_{\zeta}^{t} \left[ \frac{\partial T_f}{\partial t'} \delta(z' - z_m) \right] dz' dt'.
\]

The variation \(\Delta J\) is derived after \(Bi(t')\) is perturbed by \(\Delta Bi(t')\), \(T_i\) and \(T_f\) are perturbed by \(T_i + \Delta T_i\) and \(T_f + \Delta T_f\), respectively, in Eq. (19). Subtracting from the resulting expression the original Eq. (19) and neglecting the second-order terms, we thus find:

\[
\Delta J[Bi(t')] = \int_{\zeta}^{t} \left[ \frac{\partial T_f}{\partial z'} \delta(z' - z_m) \right] dz' dt' + \int_{\zeta}^{t} \left[ \frac{\partial T_f}{\partial t'} \delta(z' - z_m) \right] dz' dt' + \int_{\zeta}^{t} \left[ \frac{\partial T_f}{\partial z'} \delta(z' - z_m) \right] dz' dt'.
\]

The adjoint problem is different from the standard initial value problem in that the final time condition at time \(t' = t\) is specified instead of the customary initial condition at time \(t' = 0\). However, this problem can be transformed to an initial value problem by the transformation of the time variable as \(\tau' = t' - t\). Then the adjoint problem can be solved by the same method as the direct problem.

Finally the following integral term is left:

\[
\Delta J = \int_{\zeta}^{t} \left[ \frac{1}{4} \left[ T_f(0,t') - T_f(0,t) \right] \left[ \lambda_i(0,t') - \lambda_f(0,t') \right] \right] \Delta Bi(t') dt'.
\]
From the definition used in Ref. [13], we have:

$$\Delta t = \int_t^{t+1} f'(t') \Delta B(t') \, dt',$$

where \( f'(t') \) is the gradient of the functional \( J(B(t)) \). A comparison of Eqs. (29) and (30) leads to the following form:

$$f'(t') = \frac{1}{4} \left[ \left( T_{r}^{}(0,t') - T_{f}^{}(0,t') \right) \left( N_{r}^{}(0,t') / k_{B} - N_{f}^{}(0,t') \right) \right].$$

(31)

2.5. Conjugate gradient method for minimization

The following iteration process based on the conjugate gradient method is now used for the estimation of \( B(t') \) by minimizing the above functional \( J[B(t')] \):

$$B^{K+1}(t') = B^K(t') - \beta^K p^K(t'), \quad K = 0, 1, 2, \ldots,$$

(32)

where \( \beta^K \) is the search step size in going from iteration \( K \) to iteration \( K + 1 \), and \( p^K(t') \) is the direction of descent (i.e., search direction) given by:

$$p^K(t') = f^K(t') + \gamma^K p^{K-1}(t'),$$

(33)

which is the conjugation of the gradient direction \( f^K(t') \) at iteration \( K \) and the direction of descent \( p^{K-1}(t') \) at iteration \( K - 1 \). The conjugate coefficient \( \gamma^K \) is determined from:

$$\gamma^K = \frac{\int_{t}^{t+1} \left[ f^K(t') \right]^2 \, dt'}{\int_{t}^{t+1} \left[ f^{K-1}(t') \right]^2 \, dt'} \quad \text{with} \quad \gamma^0 = 0.$$  

(34)

The convergence of the above iterative procedure in minimizing the functional \( J \) is proved in Ref. [14]. To perform the iterations according to Eq. (32), we need to compute the step size \( \beta^K \) and the gradient of the functional \( f^K(t') \).

The functional \( J[B^{K+1}(t')] \) for iteration \( K + 1 \) is obtained by rewriting Eq. (10) as:

$$J[B^{K+1}(t')] = \int_{t}^{t+1} \left[ (B^K(t') - \beta^K p^K(t')) - Y' \left( z_{m}^{}, t' \right) \right]^2 \, dt'$$

(35)

where we replace \( B^{K+1} \) by the expression given by Eq. (32). If temperature \( T_{r}^{}(B^K - \beta^K p^K) \) is linearized by a Taylor expansion, Eq. (35) takes the form:

$$\int_{t}^{t+1} \left[ T_{r}^{}(B^K) - \beta^K T_{r}^{}(p^K) - Y' \left( z_{m}^{}, t' \right) \right]^2 \, dt'$$

(36)

where \( T_{r}^{}(B^K) \) is the solution of the direct problem at \( z' = z_{m}^{} \) by using estimated \( B^K(t') \) for exact \( B(t') \) at time \( t' \). The sensitivity function \( \Delta T_{r}^{}(p^K) \) are taken as the solution of Eqs. (11)–(18) at the measured position \( z = z_{m}^{} \) by letting \( \Delta B = p^K \). The search step size \( \beta^K \) is determined by minimizing the functional given by Eq. (36) with respect to \( \beta^K \). The following expression can be obtained:

$$\beta^K = \int_{t}^{t+1} \frac{\Delta T_{r}^{}(p^K) \left[ T_{r}^{}(B^K) - Y' \left( z_{m}^{}, t' \right) \right] \, dt'}{\int_{t}^{t+1} \left[ \Delta T_{r}^{}(p^K) \right]^2 \, dt'}.$$  

(37)

2.6. Stopping criterion

If the problem contains no measurement errors, the traditional check condition specified as:

$$J[B^{K+1}] < \eta,$$

(38)

where \( \eta \) is a small specified number, can be used as the stopping criterion. However, the observed temperature data contains measurement errors; as a result, the inverse solution will tend to approach the perturbed input data, and the solution will exhibit oscillatory behavior as the number of iteration is increased [14]. Computational experience has shown that it is advisable to use the discrepancy principle for terminating the iteration process in the conjugate gradient method. Assuming \( T_{r}^{}(z_{m}^{}, t') = Y' \left( z_{m}^{}, t' \right) + \sigma \), the stopping criteria \( \eta \) by the discrepancy principle can be obtained from Eq. (10) as:

$$\eta = \sigma^2 \gamma^0.$$  

(39)

where \( \sigma \) is the standard deviation of the measurement error. Then the stopping criterion is given by Eq. (38) with \( \eta \) determined from Eq. (39).

3. Results and discussion

The objective of this article is to validate the present approach when used in estimating the unknown time-dependent thermal contact resistance at the interface of a strip (the pad) and a semi-infinite foundation (the disc) during a sliding contact accurately with no prior information on the functional form of the unknown quantities, a procedure called function estimation. In the present study, we consider the simulated exact value of \( B(t') \) over the time period \( t = 0 \) to 1 as:

$$B(t') = \left( 1 - e^{-10t} \right).$$  

(40)

The material of the foundation is assumed being cast iron ChHMHk (\( k_i = 51 \text{ Wm}^{-1} \text{K}^{-1} \) and \( \alpha = 1.4 \times 10^{-5} \text{m}^2 \text{s}^{-1} \)), while the material of the strip is ceramic–metal FMK-11 of thickness \( d = 5 \text{ mm} \), (\( k_s = 343.3 \text{ Wm}^{-1} \text{K}^{-1} \) and \( \alpha = 1.52 \times 10^{-5} \text{m}^2 \text{s}^{-1} \)). The Biot’s number at the strip surface is taken as \( Bi = 1.0 \) in this study [5]. A single thermocouple is assumed to be located at the interface \( z_{m}^{} = 0 \). In terms of the time domain, the total dimensionless measurement time is chosen as \( z = 1.0 \) and measurement time step is taken to be 0.02. Besides, the same computational procedure as in Ref. [7] is used in the numerical calculations.

In the analysis, we do not have a real experimental set up to measure the temperature \( Y(z_{m}^{}, t') \) in Eq. (10). Instead, we assume a real thermal contact resistance, \( B(t') \) of Eq. (40), and substitute the exact \( B(t') \) into the direct problem of Eqs. (1)–(8) to calculate the temperatures at the location where the thermocouple is placed. The results are taken as the computed temperature \( Y_{\text{exact}}(z_{m}^{}, t') \). Nevertheless, in reality, the temperature measurements always contain some degree of error, whose magnitude depends upon the particular measuring method employed. In order to consider the situation of measurement errors, a random error noise is added to the above computed temperature \( Y_{\text{exact}}(z_{m}^{}, t') \) to obtain the measured temperature \( Y_{\text{meas}}(z_{m}^{}, t') \). Hence, the measured temperature \( Y_{\text{meas}}(z_{m}^{}, t') \) is expressed as

$$Y'_{\text{meas}}(z_{m}^{}, t') = Y_{\text{exact}}(z_{m}^{}, t') + \varepsilon,$$

(41)

where \( \varepsilon \) is a random variable within \(-2.576 \text{ to } 2.576 \) for a 99% confidence bounds, and \( \sigma \) is the standard deviation of the measurement. The measured temperature \( Y'_{\text{meas}}(z_{m}^{}, t') \) generated in such way is the so-called simulated measured temperature.

Fig. 2 shows the estimated values of the unknown function \( B(t') \), obtained with the initial guesses \( B_0 = 0.0 \), temperature measurement taken at \( z_{m}^{} = 0.0, \) and measurement error of deviation \( \sigma = 0.0 \) and 0.01, respectively. These results confirm that the estimated results are in very good agreement with those of the exact values. For a temperature of unity and 99% confidence, that standard deviation, \( \sigma = 0.01, \) corresponds to measurement error of 2.58%. The results in Fig. 2 also demonstrate that, for the cases considered in this study, an increase in the measurement error does not cause obvious...
deterioration on the accuracy of the inverse solution. Meanwhile, in order to investigate the influence of measurement location upon the estimated results, Fig. 3 illustrates the estimated unknown function $B_i(t^*)$, with temperature measurement taken at $z_m^* = -0.1$. Here, the initial guesses $B_i^0 = 0.0$ and measurement error $\sigma = 0.00$, and satisfactory results are still returned which has proved that different measurement locations pose no influence on the accuracy of the present inverse method.

The estimated temperature distributions in the strip and semi-infinite foundation for $t^* = 0.20, 0.50, \text{ and } 0.80$, respectively, are demonstrated in Fig. 4. The results in Fig. 4 are obtained with the initial guesses $B_i^0 = 0.0$, temperature measurement taken at $z_m = 0.0$, and measurement error of deviation $\sigma = 0.00$. These results confirm that the estimated temperature values are in very good agreement with those of the exact values for the case considered in this study. It can be found in Fig. 4 that, overall, the temperature rises rapidly at the interface as a consequence of the rapid rise of its internal energy by heat generation, but it drops sharply as the distance from the interface increases.

In order to demonstrate the capability of the presented methodology in obtaining an accurate estimation no matter how complex the unknown function is, we consider another case of $B_i(t^*)$ with the following form:

$$B_i(t^*) = \begin{cases} 
5t^*/0.6, & \text{for } 0 \leq t^* \leq 0.6, \\
5, & \text{for } 0.6 \leq t^* \leq 1.
\end{cases} \quad (42)$$

Fig. 5 shows the estimated results of $B_i(t^*)$, obtained with the initial guesses $B_i^0 = 0.0$, temperature measurement taken at $z_m = 0.0$, and measurement error of deviation $\sigma = 0.00$. It can be found in Fig. 5 that an excellent estimation still can be obtained with this complex unknown function.
4. Conclusion

An inverse algorithm based on the conjugate gradient method and the discrepancy principle was successfully applied to estimate the unknown time-dependent thermal contact resistance for the tribosystem consisting of a semi-infinite foundation and a plane-parallel strip sliding over its surface, while knowing the temperature history at some measurement locations. Subsequently, the temperature distributions in the system can be calculated. Numerical results confirm that the method proposed herein can accurately estimate the time-dependent thermal contact resistance and temperature distributions for the problem even involving the inevitable measurement errors.

References