An analytical solution of the stiffness equation for linear guideway type recirculating rollers with arbitrarily crowned profiles

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The manuscript was received on 5 June 2008 and was accepted after revision for publication on 24 November 2008.

DOI: 10.1243/09544062JMES1208

Abstract: This article derives the stiffness equation for linear guideway type (LGT) recirculating rollers with arbitrarily crowned profiles. Recirculating rollers compressed between a carriage and a profile rail are simulated as rollers compressed between two plates, and each roller is divided into three parts: two crowned and one cylindrical. The superposition method is introduced to obtain the stiffness equation for a crowned roller, for which rollers with circular, quadratic, cubic, fourth-order power, and exponential profiles were analysed. A model of discrete normal springs is used to obtain the stiffness equation for LGT recirculating rollers. The results reveal that a higher-order power can make an LGT with higher stiffness. The stiffness of the exponential profile is close to that of the fourth power profile. Therefore, recirculating rollers with an exponential profile, a small crowned depth, and a large crowned length seem to be optimal, obtaining a higher LGT stiffness and a lower edge stress concentration.

Keywords: stiffness equation, linear guideway, roller, arbitrarily crowned profiles, contact

1 INTRODUCTION

The linear motion roller guideway shown in Fig. 1 has many advantages compared to ball guideways [1] and conventional sliding guides, such as flat ways and V-ways [2]. For instance, the ultimate loading of the roller guideway can be larger than that of a ball guideway, making lubrication more efficient so that the abrasion of linear motion roller bearings is less than that of sliding guides. Linear motion roller bearings also have no stick slip. Linear motion roller guideways are widely used instead of ball guideways and sliding guides in heavy CNC machining centres, grinding machines, precision heavy-duty X–Y tables, and thin film transistor (TFT)-liquid crystal display (LCD) transport systems [3].

In recent years, the speed of machines using linear motion rolling bearings has increased, making the sound and vibration of linear motion rolling bearings contribute to the serious problem of noise and vibrations of these machines [4, 5]. Kasai et al. [6, 7] found that carriages of preloaded recirculating linear roller bearings were moved periodically with the roller passing frequency due to the roller circulation. Ye et al. [8] carried out a modal analysis for carriages of linear guideway type (LGT) recirculating linear roller bearings under stationary conditions and pointed out the existence of the rigid-body natural vibrations of the carriage. Schneider [9] developed a theory for natural vibrations of an LGT recirculating linear ball bearing with a 45° contact angle. The stiffness equation for a linear motion roller guideway with recirculating rollers, which are a combination of straight and curved profiles, was not developed in the mentioned studies.

The stiffness of the roller–carriage and roller–rail is often used to determine the fatigue life and the dynamic behaviour of the linear motion roller guideway. Thus, the stiffness analyses of rollers are important and necessary. Generally, this problem can be solved using the finite-element method (FEM); however, the FEM requires a complicated mesh and a time-consuming contact analysis. An alternative is to develop an analytical method that evaluates the roller stiffness efficiently and easily. It is important to reduce edge stresses on the roller contact surface and to obtain a substantially uniform contact stress distribution. This has been attained primarily by using specially crowned profiles for the contact surface [10]. Nayak [11] presented a method to calculate the pressure between elastic bodies having a
slender area of contact and arbitrarily profiles. He used Lundberg’s equation [12] in his study and suggested that the combined compressive deformation of both bodies (the stiffness between elastic bodies) should be used to construct a compatible deformation equation. Horng et al. [13] demonstrated a stiffness equation for the circularly crowned roller compressed between two plates. To obtain the optimum roller contact surface, the stiffness equation was modified to describe the stiffness other than the circularly crowned roller [14, 15].

In the present study, a stiffness equation for LGT recirculating rollers with arbitrarily crowned profiles is explored. Recirculating rollers compressed between a carriage and a profile rail are simulated as rollers compressed between two plates, and the superposition method is introduced to obtain the stiffness equation for a crowned roller compressed between a carriage and a profile rail, for which rollers with circular, quadratic, cubic, fourth-order power, and exponential profiles were analysed. Finally, a model of discrete normal springs is used to obtain the stiffness equation for LGT recirculating rollers with arbitrarily crowned profiles. The results reveal that recirculating rollers with an exponential profile, a small crowned depth, and a large crowned length seem to be optimal to obtain a higher LGT stiffness and a lower edge stress concentration.

2 STIFFNESS FORMULA FOR LGT

Figure 1 shows the configuration of an LGT recirculating roller bearing. To analyse the stiffness equation, the stiffness formula for a crowned roller compressed by a carriage and a profile rail is investigated. The model and dimensions are shown in Fig. 2 and the profiles of a crowned roller are shown in Fig. 3. The crowned roller is divided into three parts, two crowned parts and one cylindrical part, in which the primary axes of the roller system, denoted by x and y, are shown in Fig. 3. Assumptions of this stiffness formula are: (a) friction is neglected because of the lubrication of the roller, (b) the depth effect of the carriage and rail can be simulated as equivalent depth, (c) the solution is

obtained for the small-strain and linear elastic conditions, (d) rollers compressed between a carriage and a profile rail are simulated as springs with parallel connection, (e) each discrete normal spring was modelled as a crowned roller compressed between two plates, (f) the normal force is applied to each contact point of the carriage and the profile rail, (g) stick and slip effects of the contact zone between rollers and profiles are neglected, (h) the preload effect of the LGT recirculating rollers is neglected, and (i) the stiffness of recirculating rollers in motion is similar to that of recirculating rollers in static.

2.1 Stiffness formula for a crowned roller compressed by a carriage and a profile rail

2.1.1 Principal radius along the x-direction for a carriage and a profile rail and crowned part \( r_{Px} \) and \( r_{Cx} \)

Holl [16] developed the deformation of a point along the line \( y = c \) due to the load on a line parallel to the
x-axis with a length of 2c and a width of dξ as

\[
\omega(x, c, z) = \left(1 + v_1\right) \frac{2cz^2}{2\pi E_1 \left[\left[(x - \xi)^2 + z^2\right]^2 + 4c^2 + z^2\right]^{1/2}} + 2\left(1 - v_1\right) \ln\left(\frac{2c + \frac{1}{2}[\left(x - \xi^2\right)^2 + 4c^2 + \frac{1}{2}z^2]}{[\left(x - \xi^2\right)^2 + z^2]^{1/2}}\right) \times p(\xi) d\xi
\]

where \(z\) and \(x\) are the coordinates shown in Fig. 1, \(v_1\) is the Poisson’s ratio of the carriage and the profile rail, \(E_1\) is the Young’s modulus of the carriage and the profile rail, 2c is the length of the non-crowned part, and \(\omega = (1 + v)/2\pi E_1(z^2/R^3) + (2(1 - v)/R)p(\xi)d\zeta\) is the line load at \(\zeta\). For the elliptical load distribution shown in Fig. 4

\[
p(\xi) = \frac{P_{\text{total}}}{\pi b c} \left(1 - \frac{\xi^2}{b^2}\right)^{1/2} \tag{2}
\]

where \(P_{\text{total}}\) is the total contact load and \(b\) is the half width of the elliptical contact load shown in Fig. 4. For a cylinder in contact with a flat plate, \(b\) can be calculated as \([9]\)

\[
b = \left[4r_{cz} \left(\frac{P_{\text{total}}}{c}\right) \left(\frac{E'}{\pi}\right)\right]^{1/2} \tag{3}
\]

where

\[
E' = \frac{2}{(E_1 - v_1^2) + (E_2 - 1 - v_2^2)} \tag{4}
\]

in which \(v_2\) is the Poisson’s ratio of the roller, \(E_2\) is the Young’s modulus of the roller, and \(r_{cz}\) is the roller radius of the non-crowned part. The displacement difference between \(x = 0\) and \(x = b\) can be expressed as

\[
\Delta_b = \int_{-b}^{b} [\omega(0, c, 0) - \omega(b, c, 0)] d\xi \tag{5}
\]

Due to the small deformation assumption, the deformed carriage and rail can be modelled by using a circular arch with the radius of

\[
r_{pz} = \frac{\Delta_b^2 + b^2}{2\Delta_b} \tag{6}
\]

2.1.2 Principal radius along the y-direction for a carriage and crowned part (\(r_{py}\) and \(r_{cy}\))

For a half-space plate, the elastic deformation induced by the load of \(p(\xi) d\xi\) can be expressed as \([14]\)

\[
\omega \rho = \frac{(1 + v_1)}{2\pi E_1} \left[\frac{2z^2}{R^3} + \frac{2(1 - v_1)}{R}\right] p(\xi) d\eta d\xi \tag{7}
\]

where \(\tilde{R} = [\left(y - \eta^2\right)^2 + \xi^2 + \frac{1}{2}z^2]^{1/2}\) and \(p(\xi)\) is the same as that shown in equation (2).

Integrating the above equation along the line \(-c \leq \eta \leq c\) gives the deflection of a point on the \(y\)-axis due to the load on a line, which is parallel to the \(y\)-axis, with a length of 2c and width of \(d\eta\)

\[
\omega_l = \frac{(1 + v_1)}{2\pi E_1} \int_{-c}^{c} \left[\frac{z^2}{R^3} + \frac{2(1 - v_1)}{R}\right] p(\xi) d\xi d\eta = \frac{(1 + v_1)}{2\pi E_1} \left(\frac{z}{R} + \frac{c - y}{\pi}\right) \left\{\frac{c - y}{\xi - y} + \frac{\left[c - y\right]^2 + \xi^2 + 2\xi^2}{\left[c - y\right]^2 + \xi^2 + 2\xi^2}\right\} p(\xi) d\xi d\eta
\]

\[
+ \frac{(1 - v_1^2)}{\pi E_1} \left\{-\frac{c - y}{\xi - y} + \frac{\left[c - y\right]^2 + \xi^2 + 2\xi^2}{\left[c - y\right]^2 + \xi^2 + 2\xi^2}\right\} p(\xi) d\xi d\eta \tag{8}
\]

The effect of carriage thickness on deformation can be accommodated in the manner used by Dowson and Higginson \([17]\) by setting \(y = 0\) and \(z = H\), where \(H\) is the equivalent thickness of the carriage and the profile rail. The plate deformation along the \(y\)-axis is calculated relative to this reference point \((y = 0, z = H)\) in the half space. Thus, the relative deformation curve between \((0, y, 0)\) and \((0, 0, H)\), due to the elliptical load from \(\xi = -b\) to \(+b\), becomes

\[
\Delta_y = \int_{-b}^{b} [\omega_l(0, y, 0) - \omega_l(0, 0, H)] d\xi \tag{9}
\]

The curve is divided into 100 segments \([14]\) and then the radius of each segment is calculated. Finally, the averaged radii, \(r_{py}\) and \(r_{cy}\), can be obtained.

In this article, the crowned profiles shown in Fig. 5 were investigated. Except for the exponential profile, the intersection point of the straight line and the profile function is set to the original point, as shown in Fig. 5.

1. Circular profile (Fig. 5(a)), for which the profile equation is

\[
t = r_{cy} - \sqrt{r_{cy}^2 - s^2} \tag{10}
\]

where \(r_{cy} = (ds^2 + dt^2)/(2dt)\). In this article, \(dt \ll ds\), \(t \cong s^2/2r_{cy}\), so \(t \cong (dt/ds^2)s^2\), which is the same as the quadratic profile. In other words,
quadratic profiles can be approximated by the circular profile when \( dt \ll ds \).

2. Cubic profile (Fig. 5(a)), for which the profile equation is

\[
t = \left( \frac{dt}{ds^3} \right) s^3
\]  

(11)

3. Fourth-order power profile (Fig. 5(a)), for which the profile equation is

\[
t = \left( \frac{dt}{ds^4} \right) s^4
\]  

(12)

4. Exponential profile (Fig. 5(b)).

Since the exponential function is always greater than zero, the intersection point of the straight line and the exponential function is assumed to be at \( s = 0 \) and \( t = dt/n \), where \( n \) is a parameter. In this study, \( n \) is set to 100. The profile equation is

\[
t = a \exp(bs)
\]

where

\[
a = \frac{dt}{n}
\]

and

\[
b = \frac{\log(n + 1)}{ds}
\]  

(13)

In the mathematical sense, the logarithm function is the inverse function of the exponential function, so the two functions are symmetric along the line \( Y = X \); thus, the smooth behaviour of the two functions are equal. The logarithm function is not included in this article. Except for the linear and exponential profiles, the tangent directions of the straight line and the crowned profile at the intersection point are equal to zero.

2.1.3 Deformation formula of two contacting elastic bodies with spherical surfaces

In this section, the deformation formula of two contacting elastic bodies with spherical surfaces is illustrated. According to Hamrock [18], the curvature sum \( R_{xy} \) and difference \( \Gamma \) are

\[
\frac{1}{R_{xy}} = \frac{1}{R_x} + \frac{1}{R_y}
\]  

(14)

\[
\Gamma = R_{xy} \left( \frac{1}{R_x} - \frac{1}{R_y} \right)
\]  

(15)

where

\[
\frac{1}{R_x} = \frac{1}{r_{Cx}} + \frac{1}{r_{Px}},
\]

\[
\frac{1}{R_y} = \frac{1}{r_{Cy}} + \frac{1}{r_{Py}}
\]

in which \( r_{Cx} \), \( r_{Cy} \), \( r_{Px} \), and \( r_{Py} \) are the principal radii calculated in section 2.1. Since the plate deformation curve in the \( x \)-direction is concave (Fig. 6), \( r_{Px} \) is negative, and the others are positive. In this study, the superposition method is used to find the stiffness of the crowned roller compressed by two plates. First, the stiffness of the non-crowned roller compressed by two plates is calculated, and the edge deformation of the plate is assumed to be a curved surface with two principal radii, \( r_{Px} \) and \( r_{Py} \). Then, the crowned part compressed by two plates is simulated as two contacting elastic bodies with spherical surfaces.

Hamrock [18] showed that the ellipticity parameter \( \kappa \) can be written as a transcendental equation relating the curvature differences and the elliptic integrals of the first (\( \Im \)) and second (\( \varepsilon \)) kinds as

\[
\kappa = \sqrt{\frac{2\Im - \varepsilon(1+\Gamma)}{\varepsilon(1-\Gamma)}}
\]  

(16)

where

\[
\Im = \int_0^{\pi/2} \left[ 1 - \left( 1 - \frac{1}{\kappa^2} \right) \sin^2 \phi \right]^{1/2} d\phi
\]

\[
\varepsilon = \int_0^{\pi/2} \left[ 1 - \left( 1 - \frac{1}{\kappa^2} \right) \sin^2 \phi \right]^{1/2} d\phi
\]

Finally, the maximum deformation at the centre of the contact can be written from the analysis of

![Fig. 6 Configuration of an LGT recirculating roller bearing and section AA along the longitudinal direction](image-url)
Hertz \([19]\) as
\[
\delta_c = \delta \left[ \left( \frac{9}{2\varepsilon R_{xy}} \right) \left( \frac{P}{\pi\kappa E^*} \right) \right]^{1/3}
\]
\(\text{(17)}\)

2.1.4 Stiffness superposition

The stiffness contribution \(K_n\) of the non-crowned part is obtained by Hoeprich’s formula \([20]\), which is an accurate stiffness equation verified by Horng et al. \([14]\), as follows
\[
K_n = \frac{P_n}{\delta}
\]
\(\text{(18)}\)

where \(P_n\) is the load applied on the non-crowned part.

\[
\delta = \Lambda \ln \left( \frac{4H e^{-1/[2(1-\nu)](1+(H/c)^2)^{3/2}}}{\Lambda} \right)
\]
\(\text{(19)}\)

where
\[
\Lambda = 2 \left( \frac{P_c}{\varepsilon} \right) \left( \frac{E^*}{\pi} \right)
\]
\(\text{(20)}\)

From Hertz’s equation, the stiffness contribution \(K_c\) of the crowned part can be written as
\[
K_c = \frac{P_c}{\delta}
\]
\(\text{(21)}\)

where \(P_c\) is the load on the crowned part. This load is generated from the crowned part compressed by two plates, which is simulated as two contacting elastic bodies with spherical surfaces. From the investigation conducted by Horng et al. \([13]\), the deformation of point \(A, \Delta\), on the plate surface shown in Fig. 6 is \(\sim 7/10\) of the maximum plate deformation, \(\Delta_{y=0}\). Since it is difficult to find a theoretical solution for this deformation, an empirical form shown in equation (22) is used to obtain the deformation \(\delta_c\) between the plate and crowned part of the roller
\[
\delta_c = \frac{\delta}{2} + 0.7\Delta_{y=0} = \frac{\delta}{2} - 0.3\Delta_{y=0}
\]
\(\text{(22)}\)

where \(\Delta_{y=0}\) is the plate deformation at \(y = 0\), calculated using equation (9), and \(\delta_c\) is the total deformation of the roller. According to equation (17), the load of the crowned part is obtained as follows
\[
P_c = \pi\kappa E^* \left[ \frac{2\varepsilon R}{9} \left( \frac{\delta_c}{\pi} \right) \right]^{3/2}
\]
\(\text{(23)}\)

Using the superposition method, the stiffness \(K\) of one crowned roller can be written as
\[
K = K_n + 2K_c = \frac{P_n + 2P_c}{\delta}
\]
\(\text{(24)}\)

The stiffness shown in equation (24) is not an explicit form of the total applied force; thus, the total displacement \(\delta\) is defined first. Using equations (19) and (23), one can obtain \(P_c\) and \(P_n\). Finally, equation (24) can be used to evaluate the roller stiffness. In other words, one crowned roller is modelled as a spring and the spring constant can be calculated using equation (24).

2.2 Stiffness formula for LGT

Rollers compressed between a carriage and a profile rail are simulated as springs with parallel connection, as shown in Fig. 6(a). Each discrete normal spring was modelled as a crowned roller compressed between two plates, as shown in Fig. 2. In this study, two plates are introduced to simulate the depth effect of a carriage and a profile rail. In an LGT recirculating linear roller bearing without an external force, the normal force is applied to each contact point of the raceways and the rollers in the load zone, with normal elastic deformation at each contact point. Thus, each contact point has the characteristics of a spring. Since the springs exist at an interval \(s\) of the loaded rollers, they are named ‘discrete normal springs’. The carriage is supported by the discrete normal springs in the load zone of each circuit of the recirculating rollers. The discrete normal stiffness is denoted by \(K\), as shown in Fig. 6(b). When the LGT recirculating linear roller bearing is driven at a constant velocity, each contact point of the raceways and rollers constantly changes. Moreover, the total number of the rollers in the load zone varies As a result, the location and number of the discrete normal springs change, and stiffness \(K\) also varies. These changes should be considered in the theoretical analysis of the rigid-body natural vibration of the carriage. However, it is very difficult to consider these changes from a theoretical point of view. In this article, discrete normal springs are replaced by distributed normal springs, which are continuously distributed along the length of the load zone of each roller circulation.

Finally, the normal stiffness equation for LGT recirculating rollers with arbitrarily crowned profiles, denoted by \(K_V\), can be obtained as \([5]\)
\[
K_V = (4Z_i \sin^2 \alpha)K
\]
\(\text{(25)}\)

where \(Z_i\) is the average number of rollers in the load zone in one circuit of the recirculating rollers and \(\alpha\) is the contact angle shown in Fig. 6(a).

2.3 Example case and practical application of the stiffness formula for LGT

From the practical applicative point of view, the stiffness equation for LGT recirculating rollers with arbitrarily crowned profiles can be used to calculate frequency expressions for the rigid-body natural vibration of a carriage for an LGT \([21]\). For convenience, natural vibration with the translational motion mode along the \(z\)-axis is called the vertical natural vibration.
of the carriage, the natural vibration with the rotary motion mode around the $y$-axis is called the pitching natural vibration of the carriage, and the natural vibration with the rotary motion mode around the $z$-axis is called the yawing natural vibration of the carriage. The spring constant $k$ per unit length of the distributed normal springs can be written as

$$k = \frac{Z_l K}{l_i} = \frac{K_0}{4l_i \sin^2 \alpha}$$  \hspace{1cm} (26)$$

$\phi$, $\theta$, and $\psi$ are the angular displacements of the carriage around the $x$, $y$, and $z$ axes shown in Fig. 6(a), respectively, and the frequency $f_v$ of the vertical natural vibration of the carriage, the frequency $f_p$ of the pitching natural vibration of the carriage, and the frequency $f_Y$ of the yawing natural vibration of the carriage are given by

$$f_v = \frac{\sin \alpha}{\pi} \sqrt{\frac{k l_i}{M}}$$  \hspace{1cm} (27)$$

$$f_p = \frac{\sin \alpha}{2\pi} \sqrt{\frac{k l_i^3}{3J_y}}$$  \hspace{1cm} (28)$$

and

$$f_Y = \frac{\cos \alpha}{2\pi} \sqrt{\frac{k l_i^3}{3J_z}}$$  \hspace{1cm} (29)$$

where $M$ is the mass of the carriage and $J_x$, $J_y$, and $J_z$ are the moments of inertia about the $x$, $y$, and $z$ axes, respectively.

3 ILLUSTRATION OF ANALYSES

LGT recirculating rollers with arbitrarily crowned profiles are shown in Fig. 1. Due to symmetry, rollers compressed between a carriage and a profile rail are simulated as springs in parallel connection, as shown in Fig. 6(b). Each discrete normal spring was modelled as the crowned roller compressed between two plates. Figure 2 shows this model and its dimensions. The boundary conditions of the model in Fig. 2 are: (a) $Y$-direction rollers along the leftmost surface, (b) $X$-direction rollers along the back surface, (c) the same $Z$-direction displacement applied on the top surface, (d) $Z$-direction rollers along the bottom surface, (e) contact elements between the roller and plate, and (f) free for other boundaries.

For each crowned profile type, two values for the effective thickness $H$ (20 and 100 mm), two values for the crowned length $d_s$ (0.8 and 0.08 mm), and two values of crowned $d_t$ were used (0.008 and 0.08 mm). The models were loaded incrementally by the displacement control. The total number of displacement increments in the stiffness analyses was set to 10. A
total displacement of 0.005 mm, divided into ten displacement increments, was uniformly applied to the upper surface of the LGT.

4 RESULTS AND COMPARISONS

Stiffness analyses with normal and extremely crowned dimensions and effective thickness were performed for each crowned profile. First, the average number of rollers in the load zone in one circuit of the recirculating rollers $Z_L$ was set to 22, the contact angle $\alpha$ was set to 50°, the roller radius $r_{C}$ was set to 2 mm, and the roller length, $L = 2(c + ds)$, was set to 6 mm (see Fig. 3). Poisson's ratio of the carriage–profile rail and the roller were 0.29 and 0.3, respectively. Young's modulus of the carriage–profile rail and the roller were 206 and 314 GPa, respectively.

Figure 7 shows a comparison of stiffness for the LGT recirculating rollers with arbitrarily crowned profiles. When $dt \ll ds$, the circular profile is very similar to the quadratic profile. The stiffness of LGT increases when the applied load increases. These conditions are also true for the other figures. Figure 7 indicates that a higher-order power obtains higher stiffness and that the stiffness of the exponential profile is close to the fourth power profile. This is because a higher-order power has a smooth crowned curve and results in higher edge stress. Therefore, an exponential profile seems to be an optimal solution for the LGT.

Figure 8 shows a comparison of stiffness between the normal effective depth of profile rail ($H = 20$ mm) and the extremely large effective depth of the profile rail ($H = 100$ mm). Figure 8 indicates that the effective depth of the profile rail does not significantly affect the value of stiffness. Therefore, it seems reasonable that the profile rail and carriage can be modelled as two flat plates. Figure 9 shows a comparison of stiffness between the normal crowned length of a roller ($ds = 0.8$ mm) and the small crowned depth of a roller ($dt = 0.08$ mm). Figure 9 indicates that the stiffness is similar for every crowned profile when the edge crowned length $ds$ decreases. Figure 10 shows a comparison of stiffness between the normal crowned depth of a roller ($dt = 0.008$ mm) and the large crowned depth of a roller ($dt = 0.08$ mm). Figure 10 indicates that the stiffness significantly decreases when the edge crowned depth $dt$ increases.

5 CONCLUSION

In this article, LGT recirculating rollers were modelled as discrete normal springs. Then, the stiffness
equation for an LGT with arbitrarily crowned profile rollers was developed. The crowned roller was divided into three parts: one cylindrical and two crowned. The superposition method was introduced to obtain the crowned roller stiffness. The simulation results reveal that a higher-order power can make an LGT with higher stiffness. The stiffness of the exponential profile is close to that of the fourth power profile. In conclusion, the stiffness equation for an LGT plays a pivotal role from the application point of view because it is often used to determine the fatigue life and the dynamic behaviours of the mechanical system in the design and implementation of heavy CNC machining centres. According to a deeper analysis of the simulation result, it is suggested that recirculating rollers with an exponential profile, a small crowned depth, and a large crowned length seem to be optimal to obtain higher stiffness LGT and lower edge stress concentration.

ACKNOWLEDGEMENT

This study was supported by the National Science Council, Republic of China, under contract number NSC 97-2622-E-168-006-CC3.

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