Vibration and Stability Analysis of Nanoscale Processing Using Atomic Force Microscopy under the Axial Force Effect*

Thin-Lin HORNG**
**Department of Mechanical Engineering,
Kun-Shan University of Technology, Tainan, Taiwan, R.O.C.
Email address: hortl@mail.ksu.edu.tw

Abstract
An analytical solution to the flexural vibration problem during a nanomachining process using an atomic force microscope (AFM) cantilever is proposed in this paper. The modal superposition method was employed to analyze the response of an AFM subjected to a cutting force with an exciting force of an arbitrarily chosen frequency. The cutting forces were transformed into normal forces, distributed transversal-forces, and bending loading and acting on the end region of the AFM by means of the tip holder. Then, the effects of the axial force were used to solve the dynamic model. The analytical solution can be employed to evaluate vibration shape, model frequencies, and response histories of the cantilever with respect to different axial force effects and excitation frequencies. The results of this study reveal that the cutting force with a low axial effect and a moderately high excitation frequency are the best machining parameters for nanoscale processing using atomic force microscopy.

Key words: Modal Superposition Method; Flexural Vibration Problem; Nanomachining Process; Atomic Force Microscope (AFM)

1. Introduction
In this investigation, the solution of the vibration response on nanoscale processing using atomic force microscopy was obtained using the modal superposition method. The atomic force microscope (AFM) was developed for producing high-resolution images of surface structures of both conductive and insulating samples [1-3]. In addition, the AFM can be used as a cutting tool for a nanolithography work [4] and as a nanoindentation tester for evaluating mechanical properties [5]. In recent years, the AFM has also been applied to nanolithography in micro/nano electromechanical systems (MEMS/NEMS) [6], having higher resolution capabilities than conventional optical and electron-beam lithography [7]. With the development of new nanodevices, it has become more and more difficult to conduct the fabrication processes using conventional lithography. However, AFM-based mechanical lithography has been demonstrated to be a very useful technique for machining diverse nanostructures such as semiconductors, metals, and soft materials. Hu et al [8] used mechanical AFM lithography on ultrathin Ti film. Fang et al [9] carried out several scribing experiments to study the machining characterizations of the nanolithography process using AFM, and conducted a surface analysis of nanomachining Al material using AFM. They found that the AFM cantilever should not be too soft and it should have a high resonant frequency in order to minimize sensitivity to vibration noise from buildings. A large cutting bandwidth is also desirable. The dynamic vibration response is complicated and precise analysis is difficult, yet it can influence the precision of machining in the operating process. Therefore, dynamic vibration analysis is crucial and requires further investigation. Vibration behavior depends on the excitation forces, which are composed of the transverse stress and bending stress acting on the tip holder, transmitted from the cutting force.

In recent years, there has been growing interest in dynamic responses of the AFM cantilever [10-11]. Ashhab et al.[12] analyzed the dynamic behavior of a microcantilever-sample system in order to control the cantilever-sample interaction and avoid the possibility of chaos. Based on calculations of contact radius and radiation
impedance, Yaralioglu et al. [13] proposed an algorithm to calculate the contact stiffness between an AFM tip and a layered material as a function of layer thickness. Nevertheless, it seems that distributed transversal and bending loading, applied to the end region of AFM by means of tip holder, appear to be absent from the literature.

In this paper, the flexural vibration responses of a rectangular AFM cantilever subjected to a cutting force with axial force effect, which can be arbitrarily chosen, are studied analytically using the modal superposition method. The effects of transverse stress, bending stress, and the axial force are used to solve the dynamic model. The results show that the axial force effect, resulting from the horizontal cutting force, has a significant, if not first order, effect on the vibration response. This occurs because the low vibration modes are magnified by axial forces, especially for the first mode, with only the first two modal components needing to be considered. The contributions from other modes, on contrast, are too trivial to be accounted for. Moreover, we found that the magnitude of the response were relatively small and smooth.

2. Analysis

AFM is used to machine specimens. Its cantilever is a rectangular elastic beam, as shown in Fig. 1. The cantilever has a length $L$, thickness $H$, width $a$, and a tip holder with a width of $w$ and tip length $h$. When the machining is in progress, the tip makes contact with the specimen, resulting in a vertical reaction force, $F_y(t)$, and a horizontal reaction force, $F_x(t)$, which are both functions of time $t$. Assuming that the reaction forces are on the tip end, the product of the horizontal force and the tip length can form a bending stress on the bottom surface of the cantilever. The cutting system can be modeled as a flexural vibration motion of the cantilever. The motion is a function of mode shape and natural frequency, and its transverse displacement is dependent on time and the spatial coordinate $x$ [14-15]. When the cutting forces are applied, the loads transmitted from the tip holder act on the end of the AFM, and can be modeled as the two parts shown in Fig.2, termed transverse excitation $p_t(x,t)$ and bending excitation $p_h(x,t)$, respectively.

![Figure 1. Schematic diagram of an AFM cantilever nanomaching a sample.](image)

Assuming that transverse excitation is uniformly distributed on the bottom surface of the AFM with a distance from $L - w$ to $L$, it can be written as:

$$p_t(x,t) = \frac{F_y(t)}{w}, \quad L - w \leq x \leq L$$  \hspace{1cm} (1)

The ratio between $F_x(t)$ and $F_y(t)$ can be expressed as $\frac{F_x}{F_y} = \lambda(\theta,...)$, where $\lambda$ is defined as the axial effect related to the cone shape cantilever tip $\theta$ and the other machining surface properties. The bending excitations resulting from horizontal cutting
force $F_x(t)$ act on the bottom surface of the AFM within the region from $L - w$ to $L$, and can be written as:

$$p_b(x,t) = \left[12h(2L - w - 2x)\cos\theta/\pi w^3\right]F_x, \quad L - w \leq x \leq L$$  \hspace{1cm} (2)

By summing the above two excitations, the total transverse excitation $p_t(x,t)$ can be expressed as:

$$p_t(x,t) = C(x)F_x(t), \quad L - w \leq x \leq L$$  \hspace{1cm} (3)

Where $C(x) = \frac{1 + 12h(2L - w)\cos\theta/\pi w^2 - 24h\lambda\cos\theta}{\pi w^3} x$  \hspace{1cm} (4)

![Figure 2. Schematic diagram of excitations acting on the AFM.](image)

The mode-superposition analysis of a distributed-parameter system is entirely equivalent to that of a discrete-coordinate system once the mode shapes and frequencies have been determined. This is because in both cases, the amplitudes of the modal-response components are used as generalized coordinates in defining the response of the structure. In principle, an infinite number of these coordinates are available for a distributed-parameter system since it has an infinite number of modes of vibration. Practically, however, only those modal components which provide significant contributions to the response [16-17] need be considered. Moreover, axial forces acting in a flexural element may have a very significant influence on the vibration behavior of the member, resulting in modifications of both frequencies and mode shapes.

Considering a prismatic member with uniform properties along its length, the partial differential equation of motion for the elementary case of beam flexure with an axial force can be written as:

$$EI \frac{\partial^4 v(x,t)}{\partial x^4} + N(t) \frac{\partial^2 v(x,t)}{\partial x^2} + m \frac{\partial^2 v(x,t)}{\partial t^2} = p(x,t)$$  \hspace{1cm} (5)

where $v(x,t)$ is the transversal displacement response, $N(t) = F_x(t)$ is the axial force, $p(x,t)$ is the transversal excitation, $EI$ is the flexure stiffness, and $m$ is the mass per unit length. The essential operation of the mode-superposition analysis is the transformation from the geometric displacement coordinates to the modal-amplitude or normal coordinates. For a one-dimensional system, this transformation is expressed as:

$$v(x,t) = \sum_{n=1}^{\infty} \Omega_n(x)Y_n(t) = \sum_{n=1}^{\infty} q_n(x,t)$$  \hspace{1cm} (6)
where $q_n(x,t)$ is the response contribution of the $n$-th mode, $Y_n(t)$ is normal coordinate and $\Omega_n(x)$ is the $n$-th mode shape of AFM.

Making the determinant of the square matrix equal zero [18], the frequency equation is given as:

$$\delta^2 + 2\delta^3 \varepsilon^2 \cos \delta \cosh \varepsilon L + \varepsilon^4 \delta + (\delta^4 \varepsilon - \varepsilon^3 \delta^2) \sin \delta \sinh \varepsilon L = 0$$  \hspace{1cm} (7)

where

$$\delta = \left[ d^2 + \frac{g^2}{4} \right]^{1/2} + \frac{g^2}{2}, \quad \varepsilon = \left[ d^2 + \frac{g^2}{4} \right]^{-1/2} - \frac{g^2}{2}, \quad \text{and} \quad g^2 \equiv \frac{N}{EI}$$  \hspace{1cm} (8)

The solution of this transcendental equation provides the values of $\alpha L$ which represent the frequencies of vibration of the cantilever beam, and allows the mode-shape expression of Eq. (18-15) to be written in the form

$$\Omega(x) = \cos \delta x - \cosh \varepsilon L - \frac{\delta^2}{\delta^2 \sin \delta + \varepsilon \sinh \delta L} (\sin \delta x - \delta \sinh \delta L)$$  \hspace{1cm} (9)

Substituting the frequency-equation roots for $\alpha L$, respectively, into this expression, one obtains the corresponding mode-shape functions.

Equation (7) is simply a statement that any physically permissible displacement pattern can be made up by superposing appropriate amplitudes of the vibration mode shapes for the structure. Substituting Eq.(6) into Eq.(5) and using orthogonality conditions, we have:

$$M_n \ddot{Y}_n(t) + (G_n F_x(t) + \Omega_n^2 M_n) Y_n(t) = P_n(t)$$  \hspace{1cm} (10)

where $\Omega_n$ is the $n$-th modal natural frequency of the AFM and is given by [19]:

$$\Omega_n(t) = (\alpha_n L)^2 \sqrt{\frac{EI}{mL^4}}$$  \hspace{1cm} (11)

$M_n$ and $P_n$ are the generalized mass and generalized load of the $n$-th mode, given by:

$$M_n(t) = \int_0^L \Omega_n(x)^2 m \, dx$$  \hspace{1cm} (12)

$$P_n(t) = \int_0^L \Omega_n(x) p_n(x,t) \, dx$$  \hspace{1cm} (13)

$$G_n(t) = \int_0^L \Omega_n(x) \frac{d^2 \Omega_n(x)}{dx^2} \, dx$$  \hspace{1cm} (14)

Using Eq.(3) and Eq.(4), we can get

$$P_n(t) = C_n F_y(t)$$  \hspace{1cm} (15)

where

$$C_n = \int_0^L \Omega_n(x) \left( \frac{1 + 12h \lambda (2L - w) \cos \theta / \pi w^2}{w} - \frac{24h \lambda \cos \theta}{\pi w^2} \right) \, dx$$

Then, the Normal-Coordinate Response Equation, which is exactly the same equation as that considered for the discrete-parameter case, can be solved.

$$\ddot{Y}_n = \frac{C_n}{M_n} F_y - \left( \frac{G_n F_x + \Omega_n^2}{M_n} \right) Y_n = f(t,Y_n)$$  \hspace{1cm} (16)

and
\[ f(t, Y_n) = \frac{C}{M_s} F_y(t) - \left( \frac{G F_y}{M_s} + \Omega^2 \right) Y_n(t) \]  

Assuming a zero initial condition, which means that \( v(x,0) = 0 \) and \( \dot{v}(x,0) = 0 \), and provided that the cutting force \( F_y(t) \) is known, Störmer’s rule [18] is introduced to solve the above second-order system where the derivative does not appear on the right-hand side, The solution of generalized coordinate \( Y_n(t) \) can be obtained.

3. Stability Analyses

Retaining a constant axial force \( N \), Eq.(9) can be used to find the static buckling axial force. For the nonvibrating case, where \( \Omega = 0 \) so that \( \alpha = 0, \delta = g \), and \( \varepsilon = 0 \), and assuming that the transversal displacement at the free end of the AFM is \( \Delta \), the complete solution can be expressed as:

\[ \bar{\phi}(x) = D_1 \cos gx + D_2 \sin gx + \Delta \]

where \( D_1 \) and \( D_2 \) are the constants of integration. Using the boundary condition, we get:

\[ \Delta \cos gL = 0 \]

From this equation, one can conclude that either \( \Delta = 0 \) or \( \cos gL = 0 \). If \( \Delta = 0 \), there is no displacement of the AFM and the trivial solution is obtained—the AFM remains straight and the transversal displacement of the AFM does not occur. Therefore, the buckling equation is satisfied when \( \cos gL = 0 \) or

\[ gL = \frac{n \pi}{2} \quad n = 1, 3, 5, 7, \ldots \]  

Using the expression \( g^2 \equiv \frac{N}{EI} \), the lowest critical value of \( N_{cr} \) is obtained by substituting \( n = 1 \) into the equation:

\[ N_{cr} = \frac{\pi^2 EI}{4L^2} \]

4. Results and Discussion

The main purpose of this study was to analyze flexural vibration responses in nanoscale processing using atomic force microscopy. The modal superposition method was employed to analyze the response of the AFM subjected to an arbitrarily chosen cutting force. The cutting force was transformed into the distributed loading applied to the end of the AFM by means of tip holder. The effects of transversal and bending stresses were used to solve the dynamic model. In order to demonstrate the validity of the analytical solution, numerical computations were performed. Provided that the cutting force \( F_y(t) \) is a series of harmonics, \( F_y(t) \) can be written as:

\[ F_y(t) = \sum_{j=1}^{m} F_j \sin(i \omega_j t) \]  

where \( \omega_j \) are the \( j \)-th natural frequency with zero axial effect, and are set for the simulated value of the excitation frequency of the vertical cutting force. The geometric and material parameters considered are as follows:
In this study, the non-dimensional response is used to normalized the static response, defined as $F_n L^3/(3EI)$.

Firstly, a comparison between the first four vibration modes, without and with axial force, was conducted to investigate the axial effect, as shown in Fig.3. The results reveal that the low vibration mode is strongly affected by axial forces, especially in the first mode. This is because the shape of the first mode is monotonously increases from the fixed end of AFM cantilever. Figure 4 shows that the first vibration mode, which occurs when the excitation frequency $\omega_1$ acts on the AFM, becomes large when the axial effect increases. The reason is that the large axial force causes the 1-st vibration mode to deform severely. Nevertheless, Fig.4 also implies that when the excitation frequency $\omega_3$ acts on the AFM and the axial effect is large, the shape of first mode is similar to the third mode. This means that excitation frequency has much effect on the first mode when axial effect is large. This result is also valid to the 2-nd mode as shown in Fig.5.

In order to investigate the effects of each mode on the response, the response contribution of the first four modes of two excitation frequencies $\omega_1$ and $\omega_3$ respectively, are plotted in Fig.6. As expected, only 2 modal components need be considered, with the first modal being the most influential to the response. The response histories at the end point of the AFM between the cutting forces $F_j(t)$ with various exciting frequencies $\omega_n$, are shown in Fig.7. The results indicate that excitation frequencies have little effect on the transversal response at the endpoint of the AFM under a low axial effect. However, the responses are strongly affected by the large axial force effect, especially at small excitation frequencies. Figure 8 shows the response histories at the end point of an AFM for different axial effects when the excitation frequencies are $\omega_1$ and $\omega_3$ respectively. The results reveal that the responses are relatively small and smooth when the axial effect is small and the excitation frequency is large. However, a zigzag response occurs when the axial effect and the excitation frequency simultaneously become very large. In other words, the possibility of chaos will occur and this machining state should be avoided to maintain the machining accuracy. Therefore, the cutting force with a low axial effect and a moderately high excitation frequency are the best machining parameters for nanoscale processing using atomic force microscopy.
Figure 3. Comparison between the first four vibration modes for an AFM cantilever, as a magnitude with and without the axial force respectively.

Figure 4. Comparison of the 1-st vibration mode between the different axial effects when the excitation frequencies are \( \omega_1 \) and \( \omega_3 \) respectively.
Figure 5. Comparison of the 2-nd vibration mode between the different axial effects when the excitation frequencies are $\omega_1$ and $\omega_3$ respectively.

Figure 6. Comparison of the response contribution between the first four modes at the end point of the AFM when the excitation frequencies are $\omega_1$ and $\omega_3$ respectively. The axial effect is equal to 0.62.
5. Conclusion

The modal superposition method was successfully applied to an AFM-based nanomachining process with the axial force effect to determine flexural vibration responses. The analytical solution can be employed to evaluate vibration shape, model frequencies, and response histories of the cantilever with respect to different axial force effects and excitation frequencies. As expected, the vibration magnitudes are strongly affected by the axial forces because low vibration modes are magnified by axial forces, especially for the first mode. Only the first two modal components, which contribute most to the response,
need be considered. The contributions from other modes, in contrast, are too trivial to be accounted for. Moreover, the magnitude of response was relatively small and smooth because the axial force effect decreases when the excitation frequencies approach a moderately high value. This means that a cutting force with a low axial effect and a moderately high excitation frequency are the best machining parameters for nanoscale processing using atomic force microscopy.

Acknowledgment

This work was supported by the National Science Council, Taiwan, Republic of China, under grant numbers NSC-93-2212-E-168-018 and NSC 95-2622-E-168-015-CC3.

References