Closed-form solutions for the frequency shift of V-shaped probes scanning an inclined surface

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Received 3 January 2005; received in revised form 18 August 2005; accepted 19 August 2005
Available online 26 September 2005

Abstract

The analytical method to determine the frequency shift of an AFM V-shaped probe scanning the relative inclined surface in non-contact mode is proposed. If the tip is not perpendicular to the surface plane, the lateral force to the tip will occur. Consequently, there exists a moment to the end of probe. The closed-form solution of the partial differential equation with a nonlinear boundary condition is derived. The error of transforming the distributed-masses system into lumped-masses one in the force gradient method or the perturbation method is eliminated. The dimensionless parameters are introduced for reducing the numerical transaction error. The limiting case such as a uniform or tapered beam can be obtained easily from the general system. The assessments of the force gradient method, the perturbation method and the propose method determining the frequency shift of a V-shaped probe are made. It is discovered that increasing the absolute inclined angle \( \theta \) decreases the frequency shift especially for a small tip–surface distance.

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Keywords: Closed-form solution; Frequency shift; V-shaped probe; AFM

1. Introduction

Atomic force microscopy (AFM) is now widely used for imaging the surfaces of materials from the micrometer to the subnanometer scale. The atomic force microscopy (AFM) in the frequency shift mode (FSM) has been developed as a powerful technique for obtaining atomic-scale images and also information about tip–surface interactions [1,2]. Hölscher et al. [3] investigated the frequency shift of non-contact mode by using the lumped mass model and the perturbation method. It was found that the force gradient method was effective only for the limited case of small resonance amplitude. Sasaki et al. [4] investigated the frequency shift of non-contact mode by using the lumped mass model, the Lennard–Jones potential model, and the Poincare method. Rabe et al. [5] studied vibration a uniform beam without tip mass.
The interacting force between the tip and the sample surface is simulated by a linear spring. The model is unsuitable because the real interacting force is nonlinear. Weigert et al. [6] determined the frequencies of a uniform beam without tip mass in different media. Fung and Huang [7] studied the dynamic response of a piezoelectric micro beam with the Lennard–Jones potential model by using the approximated finite element method. Sokolov et al. [8] investigated the suitability of the Lennard–Jones potential model and the imagination. Because the approximated solutions are used, the conventional dynamic AFM uses lever amplitudes of 1–100 nm. This is far larger than typical short-range interaction ranges. Thus the data interpretation becomes very difficult. Since interactions often have length ranges of only a few Angstrom, sub-Angstrom amplitudes have to be used. Thus an accurate analysis can improves greatly the studies of surface image and interaction energies and interaction forces. Moreover, because the torsional rigidity of a V-shaped probe is larger than that of a uniform beam, the tip friction resulting in the torsional deformation of a V-shaped probe is smaller. It can improve the study of surface image. Sader [9,10] and Neumeister and Ducker [11] investigated only the effective spring constant of a V-shaped beam. So far, no analytical dynamic solution has been given to the response of a V-shaped beam with a tip mass, subjected to a nonlinear interacting force because of its complexity.

This paper is the generalization of that given by Lin [12]. Lin studied the frequency shift problem of an AFM probe scanning a planar surface. In this paper, the closed-form solution for the frequency shift of an AFM V-shaped probe scanning an inclined surface is derived. The assessment of the force gradient method, perturbation method and the propose method determining the frequency shift of a V-shaped beam is made. Finally, the effects of several parameters on the frequency shift are investigated.

2. Governing equation and boundary conditions

The dynamic response of a V-shaped probe with a tip, subjected to the interatomic van der Waals force is investigated, as shown in Fig. 1:

$$\frac{\partial^2}{\partial x^2} \left\{ EI \frac{\partial^2 W}{\partial x^2} \right\} + \rho A \frac{\partial^2 W}{\partial t^2} = 0. \quad (1)$$

The boundary conditions are

- at $x = 0$:
  $$\frac{\partial W}{\partial x} = 0,$$
  $$W = 0,$$

- at $x = L$:
  $$EI \frac{\partial^2 W}{\partial x^2} = F_v \sin \theta,$$
  $$\frac{\partial}{\partial x} \left[ EI \frac{\partial^2 W}{\partial x^2} \right] - m \frac{\partial^2 W}{\partial t^2} = F_v \cos \theta, \quad (5)$$

where $m$ is the tip mass, $W$ the flexural displacement, $E$ the Young’s modulus, $x$ the coordinate along the beam, $L$ the length of the beam. $A$ and $I$ denote the cross-sectional area and the area moment of inertia, respectively. $H$ is the height of tip, $\rho$ the mass density per unit volume, $\theta$ the inclined angle of surface. In general, the interacting force between a spherical apex and a planar surface is perpendicular to the surface plane. If the tip is not perpendicular to the surface plane, the lateral force to the tip will occurs. Consequently, there exists a moment to the end of probe which is the right-handed term of Eq. (4). It should be noted that if $\theta = 0$, the system becomes a common case. The area of cross-section and the area inertia of a V-shaped beam with constant thickness $h$ are:

$$A(\xi) = \begin{cases} 2w_b h, & 0 \leq x \leq L_1, \\ hB \left( 1 - \frac{x}{L_0} \right), & L_1 \leq x \leq L, \end{cases}$$

$$I = \begin{cases} \frac{1}{6} w_b h^3, & 0 \leq x \leq L_1, \\ \frac{h^3}{12} B \left( 1 - \frac{x}{L_0} \right), & L_1 \leq x \leq L. \end{cases} \quad (6)$$

The van der Waals force [13] is:

$$F_v = -\frac{A_W R}{6D^2} (1 - \cos \theta) \quad (7)$$

where the tip–surface distance $D = D_0 - W(1)$.
between the tip of the undeformed beam and the surface of sample, \( h \) the thickness, \( R \) the tip radius.

It is well known that the orders of parameters are different greatly. There exists the numerical transaction error. In order to reducing the error, the dimensionless parameters are taken. In terms of the following dimensionless quantities:

\[
b(\xi) = \frac{E(x)I(x)}{E(0)I(0)}, \quad c_v = \frac{A_0 n R L^3}{E(0)I(0)L^3},
\]

\[
D_0 = \frac{D_0}{L_0}, \quad f_v = -c_v \left( \frac{6(D_0 - w(1, \tau) \cos \theta)}{R} \right),
\]

\[
\bar{H} = \frac{H}{L}, \quad w(\xi, \tau) = \frac{W(x, t)}{L_c},
\]

\[
\frac{W_b}{B} = \frac{1}{2} (1 - \xi_1 \xi_2), \quad \mu = \frac{m_0}{\rho(0) A(0)L},
\]

\[
\xi = \frac{x}{L}, \quad \xi_1 = \frac{L_1}{L}, \quad \xi_2 = \frac{L}{L_0},
\]

\[
\tau = \frac{t}{L_c^2} \sqrt{\frac{E(0)I(0)}{\rho(0) A(0)}}.
\]

(8)

the dimensionless governing differential equations of the system are:

\[
\frac{\partial^2}{\partial \xi^2} \left[ b(\xi) \frac{\partial^2 w}{\partial \xi^2} \right] + m(\xi) \frac{\partial^2 w}{\partial \tau^2} = 0.
\]

(9)

The dimensionless boundary conditions are:

- at \( \xi = 0 \):
  \[
  \frac{\partial w}{\partial \xi} = 0, \quad w = 0,
  \]

(10)

(11)

- at \( \xi = 1 \):
  \[
  b(1) \frac{\partial^2 w}{\partial \xi^2} = f_v(\tau) \bar{H} \sin \theta,
  \]

(12)

\[
\frac{\partial}{\partial \xi} \left[ b(\xi) \frac{\partial^2 w(\xi, \tau)}{\partial \xi^2} \right] - \mu \frac{\partial^2 w(\xi, \tau)}{\partial \tau^2} = f_v(\tau) \cos \theta,
\]

(13)

where \( L_c \) is the characteristic length which is introduced for avoiding the numerical transaction error. Let \( L_c = 10 \) nm. The probe is made of a kind of material. Consequently, the dimensionless mass per unit length \( m(\xi) \) and bending rigidity \( b(\xi) \) of the V-shaped probe are, respectively

\[
m(\xi) = \begin{cases} 1, & 0 \leq \xi \leq \xi_1, \\ \frac{1 - \xi_1 \xi_2}{1 - \xi_1^2 \xi_2^2}, & \xi_1 \leq \xi \leq 1, \end{cases}
\]

(14)

\[
b(\xi) = \begin{cases} 1, & 0 \leq \xi \leq \xi_1, \\ \frac{1 - \xi_1 \xi_2}{1 - \xi_1^2 \xi_2^2}, & \xi_1 \leq \xi \leq 1, \end{cases}
\]

It should be noted that when \( \xi_1 = 0 \), the beam becomes one with the width varied linearly with the taper ratio of \( -\xi_2 \) [14]. When \( \xi_1 = 1 \), and the beam is uniform [14].

Fig. 1. (a) Geometry and coordinate system of a micro-probe scanning an inclined surface; (b) geometry and coordinate system of a V-shaped probe.
3. Solution method

So far, no analytical dynamic solution of a V-shaped beam with a tip mass, subjected to a nonlinear interacting force has been given because of its complexity. The following simple and efficient algorithm for deriving the analytical solution of the general system is proposed. It can improve the study of surface image.

3.1. Variable of separation

The dynamic solution is assumed

\[ w(\xi, \tau) = \tilde{w}(\xi) \cos \omega \tau, \] (15)

where \( \omega \) is the dimensionless frequency, \( \Omega \sqrt{\rho(0)A(0)L^4/E(0)I(0)} \). Substituting it into the governing Eq. (9) and the boundary conditions (10)–(13), one can obtain

\[
\frac{d^2}{d\xi^2} \left\{ b(\xi) \frac{d^2 \tilde{w}}{d\xi^2} \right\} - m(\xi) \omega^2 \tilde{w} = 0, \quad \xi \in (0, 1)
\] (16)

and the associated boundary conditions:

- at \( \xi = 0 \):
  \[
  \frac{d\tilde{w}}{d\xi} = 0, \quad \tilde{w} = 0,
  \] (17)

- at \( \xi = 1 \):
  \[
  b(1) \frac{d^2 \tilde{w}}{d\xi^2} \cos \omega \tau = - \frac{c_v \tilde{H} \sin \theta}{6(D_0 - \tilde{w}(1) \cos \theta \cos \omega \tau)^2},
  \]
(19)

Multiplying Eqs. (19) and (20) by \( \cos \omega \tau \) and integrating it from 0 to the period \( T \), \( 2\pi/\omega \), Eqs. (19) and (20) become, respectively:

\[
\frac{d}{d\xi} \left[ b(\xi) \frac{d^2 \tilde{w}(\xi)}{d\xi^2} \right] \cos \omega \tau + \mu \omega^2 \tilde{w} \cos \omega \tau = - \frac{c_v \cos \theta}{6(D_0 - \tilde{w}(1) \cos \theta \cos \omega \tau)^2},
\]
(20)

Multiplying Eqs. (19) and (20) by \( \cos \omega \tau \) and integrating it from 0 to the period \( T \), \( 2\pi/\omega \), Eqs. (19) and (20) become, respectively:

\[
b(1) \frac{d^2 \tilde{w}}{d\xi^2} = \tilde{f}_v \tilde{H} \sin \theta,
\] (21a)

\[
\frac{d}{d\xi} \left( b \frac{d^2 \tilde{w}}{d\xi^2} \right) + \omega^2 \mu \tilde{w} = \tilde{f}_v \cos \theta \tag{21b}
\]

where

\[
\tilde{f}_v = \frac{-c_v \tilde{w}(1) \cos \theta}{3(D_0 - \tilde{w}^2(1) \cos^2 \theta)^{3/2}}
\]

Multiplying (21a) by \( \cos \theta \) and Eq. (21b) by \( \tilde{H} \sin \theta \), then subtracting the former from the latter, one obtains:

\[
b(1) \frac{d^2 \tilde{w}}{d\xi^2} \cos \theta - \tilde{H} \sin \theta \left[ \frac{d}{d\xi} \left( b \frac{d^2 \tilde{w}}{d\xi^2} \right) + \omega^2 \mu \tilde{w} \right]_{\xi=1} = 0.
\]
(21d)

3.2. Frequency equation

The solution of Eq. (16) can be written as:

\[
\tilde{w} = C_1 V_1(\xi) + C_2 V_2(\xi) + C_3 V_3(\xi) + C_4 V_4(\xi)
\]
(22)

where \( V_i(\xi), i = 1, 2, 3, 4 \) are the four linearly independent fundamental solutions of Eq. (16), which satisfy the following normalized condition:

\[
\begin{bmatrix}
V_1(0) & V_2(0) & V_3(0) & V_4(0) \\
V_1'(0) & V_2'(0) & V_3'(0) & V_4'(0) \\
V_1''(0) & V_2''(0) & V_3''(0) & V_4''(0) \\
V_1'''(0) & V_2'''(0) & V_3'''(0) & V_4'''(0)
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}.
\]
(23)

Substituting Eq. (22) into the boundary conditions (17)–(19), (21b) and (21d), the characteristic equation is obtained:

\[
\left( \eta_3 - \frac{\beta_3}{\beta_4} \eta_4 \right) + \frac{c_v \left( V_3(1) - \frac{\beta_3}{\beta_4} V_4(1) \right) \cos \theta}{3 \left( D_0^2 - \left( C_3 \left( V_3(1) - \frac{\beta_3}{\beta_4} V_4(1) \right) \right)^2 \cos^2 \theta \right)^{3/2}} = 0.
\] (24a)
where
\[ \eta_i = b(1) V''_i(1) + b'(1) V'_i(1) + \omega^2 \mu V_i(1), \]
\[ \beta_i = -H \sin \theta b(1) V''_i(1) + [b(1) \cos \theta - b'(1) H \sin \theta] \]
\[ \times V'_i(1) - H \sin \theta \omega^2 \mu V_i(1), \quad i = 3, 4 \quad (24b) \]

If \( \theta = 0 \), Eq. (24a) becomes:
\[
\left( \eta_3 - \frac{V''(1)}{V'_4(1)} \eta_4 \right) + \frac{c_v \left( V_3(1) - \frac{V''(1)}{V'_4(1)} V_4(1) \right)}{3 \left( D_0^2 - \left( C_2 \left( V_3(1) - \frac{V''(1)}{V'_4(1)} V_4(1) \right) \right)^2 \right)^{3/2}} = 0.
\]

(24c)

The amplitude of oscillation at the tip can be rewritten as:
\[
\bar{w}(1) = C_3 \left( V_3(1) - \frac{\beta_3}{\beta_4} V_4(1) \right).
\]

(25a)

If \( \theta = 0 \), Eq. (25a) becomes:
\[
\bar{w}(1) = C_3 \left( V_3(1) - \frac{V''(1)}{V'_4(1)} V_4(1) \right).
\]

(25b)

It is observed from Eqs. (24) that the resonant frequencies are dependent on the amplitude of oscillation at the tip. Given the amplitude, the exact corresponding frequencies can be easily determined by using the numerical method proposed by Lee and Lin [14].

3.3. Fundamental solutions

In general, because the coefficients of the differential equation (16) are discontinuous and variable, it is difficult to derive its fundamental solutions. The following method to determine the solutions is proposed.

For \( 0 \leq \xi < \xi_1 \) the governing differential equation (16) is:
\[ \frac{d^4 \bar{w}}{d\xi^4} - \omega^2 \bar{w} = 0, \]

(26)

The four linearly independent normalized fundamental solutions of Eq. (26) can be derived easily:
\[ V_{1,1}(\xi) = \frac{1}{2} (\cosh \sqrt{\omega \xi} + \cos \sqrt{\omega \xi}), \]
\[ V_{2,1}(\xi) = \frac{1}{2\sqrt{\omega}} (\sinh \sqrt{\omega \xi} + \sin \sqrt{\omega \xi}), \]
\[ V_{3,1}(\xi) = \frac{1}{2\omega} (\cosh \sqrt{\omega \xi} - \cos \sqrt{\omega \xi}), \]
\[ V_{4,1}(\xi) = \frac{1}{2\omega^{3/2}} (\sinh \sqrt{\omega \xi} - \sin \sqrt{\omega \xi}), \]

(27)

where the second subscript entitled ‘1’, describes the first section of beam. These fundamental solutions satisfy the normalized condition (23).

For \( \xi_1 < \xi < 1 \) the governing differential equation (16) is:
\[ \frac{d^2}{d\xi^2} \left( (1 - \xi_2 \xi) \frac{d^2 \bar{w}}{d\xi^2} \right) - (1 - \xi_2 \xi) \omega^2 \bar{w} = 0. \]

(28)

Its four linearly independent fundamental solutions can be derived easily as follows. Eq. (28) can be expressed in the following form:
\[ p_4(\xi) \frac{d^4 \bar{w}}{d\xi^4} + p_3(\xi) \frac{d^3 \bar{w}}{d\xi^3} + p_0(\xi) \bar{w} = 0, \]

(29)

where
\[ p_4(\xi) = a_0 + a_1 (\xi - \xi_1), \quad p_3(\xi) = b_0, \]
\[ p_0(\xi) = e_0 + e_1 (\xi - \xi_1), \]

(30a)
in which
\[ a_0 = (1 - \xi_1 \xi_2), \quad a_1 = -\xi_2, \quad b_0 = -2\xi_2, \]
\[ e_0 = -(1 - \xi_1 \xi_2) \omega^2, \quad e_1 = \xi_2 \omega^2. \]

(30b)

One assumes that the four fundamental solutions are:
\[ V_{j,2}(\xi) = \sum_{i=0}^{\infty} a_{i,j} (\xi - \xi_1)^i, \]

(31a)
where

\[ \alpha_{01} = 1, \quad \alpha_{11} = \alpha_{21} = \alpha_{31} = 0, \]
\[ \alpha_{12} = 1, \quad \alpha_{02} = \alpha_{22} = \alpha_{32} = 0, \]
\[ \alpha_{23} = \frac{1}{2}, \quad \alpha_{03} = \alpha_{13} = \alpha_{33} = 0, \]
\[ \alpha_{34} = \frac{1}{6}, \quad \alpha_{04} = \alpha_{14} = \alpha_{24} = 0 \] (31b)

These four fundamental solutions (31) satisfy the following normalized condition:

\[
\begin{bmatrix}
V_{12}(\xi_1) & V_{22}(\xi_1) & V_{32}(\xi_1) & V_{42}(\xi_1) \\
V'_{12}(\xi_1) & V'_{22}(\xi_1) & V'_{32}(\xi_1) & V'_{42}(\xi_1) \\
V''_{12}(\xi_1) & V''_{22}(\xi_1) & V''_{32}(\xi_1) & V''_{42}(\xi_1) \\
V'''_{12}(\xi_1) & V'''_{22}(\xi_1) & V'''_{32}(\xi_1) & V'''_{42}(\xi_1)
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (32)

Substituting Eqs. (30) and (31) into Eq. (29) and collecting the coefficients of like powers of \((\xi - \xi_1)\), the following recurrence formula can be obtained:

\[ \alpha_{i,k+1} = \frac{-(\alpha_{i,k} + \alpha_{i,k-1} + (k + 3)(k + 2))}{(k + 4)(k + 3)(k + 1)a_0} \] (33)

With this recurrence formula, one can generate the four exact fundamental solutions in Section 2.

Consider the following continuity conditions at the interface of two sections:

\[ \bar{w}(\xi_1^-) = \bar{w}(\xi_1^+), \quad \frac{d\bar{w}(\xi_1^-)}{d\xi} = \frac{d\bar{w}(\xi_1^+)}{d\xi}, \]
\[ \frac{d^2\bar{w}(\xi_1^-)}{d\xi^2} = \frac{d^2\bar{w}(\xi_1^+)}{d\xi^2}, \quad \frac{d^3\bar{w}(\xi_1^-)}{d\xi^3} = \frac{d^3\bar{w}(\xi_1^+)}{d\xi^3} \] (34)

Moreover, the general solution (22) can be written as:

\[ \bar{w} = \begin{cases} 
\sum_{j=1}^{4} C_j V_{j1}(\xi), & 0 < \xi < \xi_1 \\
\sum_{j=1}^{4} C_j V_{j2}(\xi), & \xi_1 \leq \xi \leq 1
\end{cases} \] (35)

Comparing of Eqs. (22) and (35) reveals that for \(0 < \xi < \xi_1\), \(V_i(\xi) = V_{i1}(\xi)\). Substituting Eqs. (22) and (35) into the continuity conditions (34), the general fundamental solutions are derived

\[ V_i(\xi) = \begin{cases} 
V_{i1}(\xi), & 0 < \xi < \xi_1 \\
\sum_{j=1}^{4} V_{j(i-1)}(\xi_1) V_{j2}(\xi), & \xi_1 < \xi < 1
\end{cases} \] (36)

4. Gradient methods

4.1. Distributed mass method

Without consideration of the effect of inclined surface, \(\theta = 0\), the van der Waals force between the tip and sample is approximated by a linear spring [5]. The spring constant is expressed by the partial derivative of the van der Waals force \(f_v\) with respect to \(z\) in the set point position \(D_0\), i.e., \(k^* = -\frac{df_v}{dz}|_{z=D_0}\). Given the van der Waals force the dimensionless spring constant is obtained:

\[ k^* = \frac{-c_v}{3D_0} \] (37)

The boundary condition (20) is replaced by:

\[ \frac{d}{d\xi} \left[ b(\xi) \frac{d^2\bar{w}(\xi)}{d\xi^2} \right] + \mu \omega^2 \bar{w} = k^* \bar{w}(1) \] (38)

The corresponding governing differential equation and boundary conditions of the system are Eqs. (16)–(19) and (38), respectively. Substituting the general solution (22) into the boundary conditions (17)–(19) and (38), the frequency equation is obtained:

\[ V_3''(1)[b(\xi) V_4''(1) + (\mu \omega^2 - k^*)V_4(1)] - V_3''(1)[b(\xi) V_4''(1) + (\mu \omega^2 - k^*)V_3(1)] = 0. \] (39)

The root of the frequency equation is the natural frequency of the system. Given the tip-sample distance, the frequencies can be easily determined by using the numerical method proposed by Lee and Lin [14]. It is observed that the effect of the amplitude of oscillation on the frequency shift is neglected in the gradient force method. When \(k^*\) is zero, the system
becomes in a free vibration of motion. In the same way, the natural frequencies of the cantilever without the influence of the interaction force between the tip and sample can be calculated.

4.2. Lumped mass method

Hölscher et al. [3] investigated the frequency shift in dynamic force microscopy with inclined angle $\theta = 0$. The distributed parameter system is simulated by a concentrated mass one. The frequency shift is derived as follows:

$$\Delta f_\theta \approx -\frac{f_0 A_H R}{2c_z 3D_0^2},$$  \hspace{1cm} (40)

where $c_z$ is the spring constant of the cantilever and $f_0$ the fundamental frequency of the cantilever without the influence of the interaction force between the tip and sample. The spring constant $c_z$ determined by using Castigliano’s second theorem is

$$c_z = \frac{1}{\frac{2L_1^3}{Ew_0h^4} + \frac{12L_0}{Eh^3B} \left[ \frac{1}{2} (L_1^2/L_2^2) + L_0(L_1/L_0 - L_2^4) \ln \frac{L_0 - L_1}{L_0 - L_2} \right]}.$$

Substituting the fundamental frequency $f_0$ determined by using the method presented in the last section and the spring constant (41) into Eq. (40), the frequency shift $\Delta f_\theta$ is obtained.

5. Perturbation method

Giessibl [15] investigated the frequency shift in dynamic force microscopy with inclined angle $\theta = 0$. The distributed parameter system is simulated by a concentrated mass one. The frequency shift is obtained by using the perturbation method as follows:

$$\Delta f_p = -\frac{f_0 A_H R}{6c_z} \frac{1}{(D_0^2 - W^2(1))^{3/2}}.$$

Substituting the fundamental frequency $f_0$ and the spring constant (41) into Eq. (42), the frequency shift $\Delta f_p$ is obtained.

6. Numerical results and discussion

To illustrate the application of the proposed method and explore the influence of the parameters on the frequency shift, the following investigations are presented.

The effects of the amplitude of tip vibration $\bar{w}(1)$ and the geometrical properties such as $\xi_1$, $\xi_2$ and the radius of tip $R$ on the frequency shift are investigated and shown in Figs. 1(b) and 2. Moreover, the comparisons of the numerical results by the proposed method and two gradient force methods [3,5] and the perturbation method [14] are made. Consider the probe made of silicon dioxide. The Young’s modulus $E = 70.3 \times 10^9$ Pa. The mass density per unit volume $\rho = 2.5 \times 10^3$ kg/m$^3$. The Young’s modulus of beam $h = 3.5$ $\mu$m. The length of beam $L = 200$ $\mu$m. The tip mass $m_t = 3.18 \times 10^{-13}$ kg. The Hamaker constant $A_H = 10^{-19}$ J [7].

The influence of the tip–surface distance on the frequency shift of three kinds of beams is investigated. The difference of the frequency shifts can be studied. Letting $\xi_1 = \xi_2 = 0$, a uniform beam is obtained. The influence of the tip–surface distance on the frequency shift of a uniform beam with $\xi_1 = \xi_2 = 0$, $R = 150$ nm and $\bar{w}(1) = 1$ nm, is shown in Fig. 2(a). Letting $\xi_1 = 0$ and $\xi_2 = 0.4$, a tapered beam is obtained. The influence of the tip–surface distance on the frequency shift of a tapered beam with $\xi_1 = 0$, $\xi_2 = 0.4$, $R = 150$ nm and $\bar{w}(1) = 1$ nm, is shown in Fig. 2(b). Letting $\xi_1 = 0.8$ and $\xi_2 = 0.4$, a V-shaped beam is obtained. The influence of the tip–surface distance on the frequency shift of a V-shaped beam with $\xi_1 = 0$, $\xi_2 = 0.4$, $R = 150$ nm and $\bar{w}(1) = 1$ nm, is shown in Fig. 2(c).

It is observed from Fig. 2(a) that when the tip–surface distance is small, the results determined by the two force gradient methods are far smaller than the presented solutions and those by the perturbation method. The errors of frequency shift determined by using the force gradient methods are satisfactory only for larger tip–surface distances. The numerical results determined by the perturbation and proposed methods are very consistent. However, Fig. 2(b and c) shows that for a tapered beam or a V-shaped beam when the tip–surface distance is small, the results determined by the perturbation method are evidently larger than those closed-form solutions. The error is due to the transforming the original distributed-masses system...
Fig. 2. Influence of the tip–surface distance \( D \) on the frequency shift of V-shaped beams. \( D = D_0 - \bar{W}(1) \), \( \theta = 0 \), \( \bar{W}(1) = 1 \) nm, \( W_b = 32.5 \) μm, \( h = 3.5 \) μm, \( L = 200 \) μm, \( A_H = 10^{-19} \) J, \( E = 70.3 \times 10^3 \) Pa, \( \rho = 2.5 \times 10^3 \) kg/m³, \( m_t = 3.18 \times 10^{-13} \) kg. (a) Present method; (b) perturbation method [15]; (c) force gradient method [5]; (d) force gradient method [3]: (a) \( \bar{W}(1) = 1 \) nm, \( \xi_1 = 0 \), \( \xi_2 = 0 \), \( R = 150 \) nm, \( f_0 = 74538.5 \) Hz; (b) \( \bar{W}(1) = 1 \) nm, \( \xi_1 = 0 \), \( \xi_2 = 0.4 \), \( R = 150 \) nm, \( f_0 = 86612.4 \) Hz; (c) \( \bar{W}(1) = 1 \) nm, \( \xi_1 = 0.8 \), \( \xi_2 = 0.4 \), \( R = 150 \) nm, \( f_0 = 77197.4 \) Hz; (d) \( \bar{W}(1) = 1 \) nm, \( \xi_1 = 0.8 \), \( \xi_2 = 0.4 \), \( R = 200 \) nm, \( f_0 = 77197.4 \) Hz; (e) \( \bar{W}(1) = 1 \) nm, \( \xi_1 = 0.8 \), \( \xi_2 = 0.4 \), \( R = 150 \) nm, \( f_0 = 77197.4 \) Hz.
into lumped-masses one before proceeding the perturbation method. Moreover, Fig. 2(a–c) reveals that the frequency shifts of a tapered beam are larger than those of a uniform beam or a V-shaped beam. It means that the precision of surface image using a tapered beam is better than that using a uniform beam or a V-shaped beam. However, because the torsional rigidity of a V-shaped beam is larger, the error of surface image due to the friction force will be smaller.

The influence of the radius of tip on the frequency shift of a V-shaped beam is investigated. Fig. 2(d) shows the influence on the frequency shift of a V-shaped beam with $\xi_1 = 0.8$, $\xi_2 = 0.4$, $R = 200$ nm and $\bar{w}(1) = 1$ nm. Comparing of those shown in Fig. 2(c and d) reveals the influence of the radius of tip on the frequency shift. It is found that the larger the radius of tip is, the larger the frequency shift greatly. It is the reason that the larger the radius of tip is, the larger the van der Waals force. However, the larger the radius of tip is, the larger the error of scanning due to surface roughness. When a rough surface is being scanned, the edges of the tail features will produce a mirror image of the side-walls of the tip rather than of the object itself. The phenomenon is called as the effect of probe-broadening [16].

The influence of the amplitude of oscillation on the frequency shift of a V-shaped beam is investigated. Fig. 2(e) shows the influence on the frequency shift of a V-shaped beam with $\xi_1 = 0.8$, $\xi_2 = 0.4$, $R = 150$ nm and $\bar{w}(1) = 5$ nm. Comparing of those shown in Fig. 2(d and e) reveals the influence of the amplitude of oscillation on the frequency shift. It is found that increasing the amplitude of oscillation decreases greatly the frequency shift.

Fig. 3 shows the influence of the hole ratio $\xi_1$ and the tip–surface distance $D$ on the frequency shift of a V-shaped beam. The ratio of geometry $\xi_2$ and the width $W_h$ are constant. When $\xi_1$ is increased, the width $B$ is increased. If $\xi_1 = 0$, the beam becomes one that its width varies linearly with a taper ratio $-\xi_2$. It is observed that decreasing the hole ratio $\xi_1$ decreases the width $B$ and increases greatly the frequency shift. However, the smaller the width $B$ is, the smaller the torsional rigidity.

Fig. 4 shows the influence of the width $W_h$ and the tip–surface distance $D$ on the frequency shift of a V-shaped beam. The ratios of geometry $\xi_1$ and $\xi_2$ are constant. When the width $W_h$ is increased, the width $B$ is increased. It is observed that increasing the width $W_h$ increases the width $B$ and decreases greatly the frequency shift. It is implied that the stiffer the root of beam is, the smaller the frequency shift.
Fig. 5 shows the influence of the inclined angle $\theta$ and the tip–surface distance $D$ on the frequency shift. The frequency shift is great at small amplitude of oscillation and tip–surface distance.

Increasing the absolute inclined angle $\theta$ decreases the frequency shift especially for a small tip–surface distance. The reason is that when the deflection of cantilever, scanning the surface with a positive inclined angle $\theta$ is downward, the moment due to the van der Waals force is against the downward motion. In the other hand, although for a negative inclined angle $\theta$, the tip moment due to the van der Waals force boosts the motion, the frequency shift is decreased still. It is because the inclined angle $\theta$ decreases the downward attractive force.

7. Conclusions

The closed-form solution for the frequency shift of an AFM V-shaped probe scanning a relative inclined surface is obtained. The comparison among the force gradient method, perturbation method and the proposed analytical method is made. It is found that the interpretation of frequency shift by using the force gradient method is unsatisfactory. For a uniform beam the numerical results determined by the perturbation method and the proposed analytical method are very consistent. However, for a tapered or V-shaped beam at a small tip–surface distance the frequency shift determined by the perturbation method is over estimated. Moreover, the effects of several parameters on the frequency shift are investigated. Several trends can be obtained as follows:

1. The frequency shift is great at small amplitude of oscillation and tip–surface distance.
2. The frequency shift of a tapered beam is greatly larger than that of a uniform or V-shaped beam.
3. Decreasing the width of beam at the root increases obviously the frequency shift.
4. Increasing the radius of tip increases obviously the frequency shift. But the error due to the effect of probe-broadening will be increased.
5. Increasing the absolute inclined angle $\theta$ decreases the frequency shift especially for a small tip–surface distance.

Acknowledgment

The support of the National Science Council of Taiwan, ROC, is gratefully acknowledged (Grant number: Nsc92-2212-E168-006).

References


