A new model for investigating the flexural vibration of an atomic force microscope cantilever

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ABSTRACT
A new model for the flexural vibration of an atomic force microscope cantilever is proposed, and a closed-form expression is derived. The effects of angle, damping and tip moment of inertia on the resonant frequency were analysed. Because the tip is not exactly located at one end of the cantilever, the cantilever is modelled as two beams. The results show that the frequency first increases with increase in angle and then decreases to a constant value for high values of the angle. Moreover, the damping is increased at lower contact positions. The tip moment of inertia is also sensitive to the resonant frequency at small values for the odd modes and large values for the even modes.

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1. Introduction

The atomic force microscope (AFM) is a powerful instrument for atomic manipulation and for producing three-dimensional high-resolution topographic images of sample surfaces for both conductors and insulators on an atomic scale [1–3]. In the contact mode, when a tip scans across a sample surface, it induces a dynamic interaction force between the tip and the surface. This dynamic interaction behaviour between the cantilever and sample is complicated, and its analysis can help to increase the resolution of surface images [4–7]. In the last two decades, much research has been done on the dynamic behaviour of the atomic force microscope [8–13] but, for convenience, some important parameters have been ignored. In practice, it is very hard to keep the cantilever parallel to the sample surface, causing an angle to inevitably develop between the cantilever and the sample. This angle affects the dynamic behaviour of the system [14]. Chang [14] was the first to analyse the sensitivity of the frequency to the angle at the contact at the end of the cantilever. However, he did not include the effects of cantilever thickness and tip moment of inertia on the contact position. The effect of damping during contact has also been ignored in other studies. Chang et al. [15] investigated the influence of normal damping using a set of springs and dashpots. They did not consider the effects of the contact position, tip moment of inertia and lateral damping. Abbasi and Karami [16] have also done some analysis on the effects of contact position and tip properties on the resonant frequency, but they did not take into account the effects of angle, damping and tip moment of inertia. Lin and Wang [17] developed an analytical model of a cantilever with an eccentric tip that was subjected to a nonlinear tip-sample surface.

A description of the cantilever as a continuous beam has been applied to AFM [18–23]. The output signal of the first mode is used to image the topography of the sample surface while the second mode is used to map changes in the mechanical, magnetic or electrical properties of the surface [24]. Moreover, the emergence of multifrequency excitation/detection schemes has caused the analysis of properties of the second mode to become relevant [24–26]. Higher harmonics have also been suggested as a way to measure Young's modulus when the tip interacts under strong repulsive forces [27–29]. Furthermore, numerical simulations have provided a rationale for using AFM schemes based on the simultaneous excitation of several higher modes [20].

In this paper, a new model for the AFM cantilever is proposed that account for most of the parameters that have been previously neglected. Because the tip is not exactly located at the end of the cantilever, the cantilever is modelled as two beams. Using a set of normal and lateral springs and dashpots, we investigate the influence of damping. The angle, contact position and tip moment of inertia are also considered in this model.

2. Analysis

The atomic force microscope cantilever developed in the present study, shown in Fig. 1, is a rectangular elastic beam that has a length L, width b and thickness t. The angle between the
cantilever and the sample surface is $z$. $L_1$ and $L_2$ are the lengths of the cantilever on the left and right sides of the tip, respectively. The cantilever has a tip near its end. Fig. 2 shows a schematic of the AFM tip. In this figure, $m_t$ and $F_t$ are the mass and moment of inertia of the tip, respectively. The AFM cantilever interacts with the sample via a normal spring $K_{n}$, a normal dashpot $C_{n}$, a lateral spring $K_{l}$ and a lateral dashpot $C_{l}$.

As shown in Fig. 1, the coordinate system of $p$ and $q$ is parallel and perpendicular to the plane, respectively, and the coordinate system of $x$ and $y$ is parallel and perpendicular to the beam axis, respectively. Because the initial deflection, $y(x)$, does not have any effect on the governing equations, one can neglect the axial load and write the governing equation as follows:

$$\frac{\partial^2 w}{\partial x^2} + \rho A \frac{\partial^2 w}{\partial t^2} = 0 \quad -L_1 \leq x \leq 0, \quad 0 \leq x \leq L_2$$

(1)

where $A$ and $l$ are the area and moment of inertia of the cross section, respectively, and $E$ is the modulus of elasticity. The boundary conditions are derived as follows:

$$w(-L_1, t) \frac{\partial w}{\partial x} \bigg|_{x = -L_1} = 0$$

(2)

$$-\frac{\partial^2 w}{\partial x^2} \bigg|_{x = -L_1} = \frac{\partial^2 w}{\partial x^2} \bigg|_{x = L_2} = 0$$

(3)

The continuity conditions are written as follows:

$$w(0^-) = w(0^+) \frac{\partial w}{\partial x} \bigg|_{x = 0^-} = \frac{\partial w}{\partial x} \bigg|_{x = 0^+}$$

(4)

$$\frac{\partial^2 w}{\partial x^2} \bigg|_{x = 0^-} = -\frac{\partial^2 w}{\partial x^2} \bigg|_{x = 0^+}$$

(5)

$$Y(-L_1) = \frac{dy}{dx} \bigg|_{x = -L_1} = 0$$

(6)

$$Y(0) = Z(0)$$

(7)

$$\frac{d^2 Z(L_2)}{dx^2} = 0, \quad \frac{dZ(L_2)}{dx} = 0$$

(8)

$$\frac{d^2 Z(L_2)}{dx^2} = \frac{d^2 Z(L_2)}{dx^2} \bigg|_{x = 0} = 0$$

(9)

$$\frac{dY(x)}{dx} \bigg|_{x = 0} = \frac{dZ(x)}{dx} \bigg|_{x = 0} = 0$$

(10)

$$\frac{d^2 Y(x)}{dx^2} = 0, \quad \frac{d^2 Z(x)}{dx^2} = 0$$

(11)

$$-\frac{\partial^2 w}{\partial x^2} \bigg|_{x = 0} = -\frac{\partial^2 w}{\partial x^2} \bigg|_{x = 0} = 0$$

(12)

$$\frac{d^2 Y(x)}{dx^2} = \frac{d^2 Z(x)}{dx^2} \bigg|_{x = 0} = 0$$

(13)

$$-\frac{\partial^2 w}{\partial x^2} \bigg|_{x = 0} = -\frac{\partial^2 w}{\partial x^2} \bigg|_{x = 0} = 0$$

(14)

$$\frac{d^2 w}{dx^2} \bigg|_{x = 0} = \frac{d^2 w}{dx^2} \bigg|_{x = 0} = 0$$

(15)
Substituting the boundary conditions of Eqs. (8) and (9) into Eq. (14), the general solution can be simplified as

$$Y(x) = C_1 \sinh(x + L_1) + C_2 \cosh(x + L_1)$$

Inserting the continuity conditions of Eqs. (10) and (11) into Eq. (15) gives

$$C_1 = F_1 D_1 + F_2 D_2$$

and used in the analysis as follows:

$$F_1 = \left[ \left( \frac{\sinh\gamma - \sin\gamma}{1 + \frac{C_p}{C_0}} \right) + \frac{\sinh\gamma - \sin\gamma}{1 + \frac{C_p}{C_0}} \right] / 2$$

$$F_2 = \left[ \left( \frac{\sinh\gamma - \sin\gamma}{1 + \frac{C_p}{C_0}} \right) + \frac{\sinh\gamma - \sin\gamma}{1 + \frac{C_p}{C_0}} \right] / 2$$

$$G_1 = \left[ \left( \frac{\sinh\gamma - \sin\gamma}{1 + \frac{C_p}{C_0}} \right) - \frac{\sinh\gamma - \sin\gamma}{1 + \frac{C_p}{C_0}} \right] / 2$$

$$G_2 = \left[ \left( \frac{\sinh\gamma - \sin\gamma}{1 + \frac{C_p}{C_0}} \right) + \frac{\sinh\gamma - \sin\gamma}{1 + \frac{C_p}{C_0}} \right] / 2$$

where $C_p = L_2/L_1$, is the contact position and $\gamma = \beta l$ is the normalised wave number.

Moreover, inserting the continuity conditions of Eqs. (12) and (13) into Eq. (15) gives

$$h_1 C_1 + h_2 C_2 + h_3 D_1 + h_4 D_2 = 0$$

$$k_1 C_1 + k_2 C_2 + k_3 D_1 + k_4 D_2 = 0$$

where

$$h_1 L_1^2 = -\gamma^2 \frac{1}{(1 + \frac{C_p}{C_0})} \left( \frac{\sinh\gamma}{1 + \frac{C_p}{C_0}} + \frac{\sin\gamma}{1 + \frac{C_p}{C_0}} \right)$$

$$h_2 L_1^2 = -\gamma^2 \frac{1}{(1 + \frac{C_p}{C_0})} \left( \frac{\cosh\gamma}{1 + \frac{C_p}{C_0}} + \frac{\sin\gamma}{1 + \frac{C_p}{C_0}} \right)$$

And

$$h_3 L_1^2 = -\gamma^2 \frac{1}{(1 + \frac{C_p}{C_0})} \left( \frac{\cosh\gamma}{1 + \frac{C_p}{C_0}} - \frac{\sin\gamma}{1 + \frac{C_p}{C_0}} \right)$$

$$h_4 L_1^2 = -\gamma^2 \frac{1}{(1 + \frac{C_p}{C_0})} \left( \frac{\cosh\gamma}{1 + \frac{C_p}{C_0}} + \frac{\sin\gamma}{1 + \frac{C_p}{C_0}} \right)$$

In the above equations, $k_n = k_n/\rho L$ and $\beta_1 = k_1/\rho L^2$ are the normal and the lateral contact stiffness, respectively. $m = mL$ is the effective mass and $J = mL/\rho L$ is the effective tip moment of inertia.

Finally, using Eqs. (16)–(19), the characteristic equation of the system can be found as follows:

$$U_1 U_4 - U_2 U_3 = 0$$

where

$$U_1 = h_1 F_1 - h_2 G_1 + h_3, \quad U_2 = h_1 F_2 - h_2 G_2 + h_4$$

And

$$U_3 = k_1 F_1 - k_2 G_1 + k_3, \quad U_4 = k_1 F_2 - k_2 G_2 + k_4$$

Therefore, the relation between frequency and wave number is given as

$$f = \frac{\gamma}{2\pi L} \sqrt{\rho \frac{L}{J}}$$

For convenience, a relative shift of the wave number is defined and used in the analysis as follows:

$$E_n = \frac{\gamma_2 - \gamma_1}{\gamma_1} \times 100\%$$
3. Results and discussion

The aim of this article was to study the effect of the angle between the cantilever and the sample and the effect of damping, as well as the effect of the moment inertia of the tip, on the resonant frequency of the vibration modes. A good analysis must consider all of the parameters simultaneously. In order to determine the effects of various parameters on the resonant frequency, we considered the geometric and material parameters of the cantilever listed in Table 1.

First, the influence of the angle on the resonant frequency is considered for $C_n = 1e-9$, $C_l = 1e-8$, $b_n = 30$, and $b_l = 0.9 b_n$. Figs. 3–5 show the effect of angle on the resonant frequency as a function of contact position for the first three modes.

As can be seen from Figs. 3–5, increase in the angle or order of the mode also increases the resonant frequency of the cantilever. Furthermore, the slope of the graph increases with increase in angle. To further investigate the effect of angle on the resonant frequency, the normalised wave number, $\gamma$, as a function of angle, $\alpha$, for the first mode is depicted in Fig. 6.

The normalised wave number increases with increase in angle until the angle reaches about $15^\circ$. After this point, the normalised wave number decreases with increase in angle and reaches a constant value for $\alpha \geq 30$. This means that the frequency is a maximum for $\alpha \approx 15^\circ$.

The effect of damping on the resonant frequency is investigated in Fig. 7 for $b_l/b_n = 0.5$, $\alpha = 0$, $C_p = 0$ and the following two cases:

1. $C_n = 1e-6$, $C_l = 1e-7$.

2. $C_n = 0$, $C_l = 0$.

It is important to note in Fig. 6 that the greatest sensitivity of the damping on the frequency occurs at a small normal stiffness of the system, when the sensitivity of the stiffness on the frequency is also small [40]. By increasing the system stiffness incrementally, the effect of the damping decreases such that at high values of the normal stiffness, about $k > 100$, the damping of the system can be neglected.

Table 1
Parameter for an AFM cantilever.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus $E$ (GPa)</td>
<td>170</td>
</tr>
<tr>
<td>Density $\rho$ (kg/m$^3$)</td>
<td>2330</td>
</tr>
<tr>
<td>Length $L$ ((\mu)m)</td>
<td>300</td>
</tr>
<tr>
<td>Thickness $t$ ((\mu)m)</td>
<td>2</td>
</tr>
<tr>
<td>Width $b$ ((\mu)m)</td>
<td>50</td>
</tr>
</tbody>
</table>

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2. $C_n = 0$, $C_l = 0$.

It is important to note in Fig. 6 that the greatest sensitivity of the damping on the frequency occurs at a small normal stiffness of the system, when the sensitivity of the stiffness on the frequency is also small [40]. By increasing the system stiffness incrementally, the effect of the damping decreases such that at high values of the normal stiffness, about $k > 100$, the damping of the system can be neglected.
The influence of the tip moment of inertia on the resonant frequency for the first four modes is depicted in Figs. 8–11 for $\beta/n = 0.5$, $\alpha = 0$, $C_p = 0$ and for the following two cases:

1. $J_f = 1.35e-2$,
2. $J_f = 0$.

It can be seen that the individual variations of the odd and even modes are the same. These figures also show that, in the odd modes, for low values of contact stiffness, the tip moment of inertia does not have any effect on the resonant frequency. However, by gradually increasing the contact stiffness of the system, to approximately $\beta/n > 10$, the influence increases. This process is different for the even modes; for small values of the contact stiffness, the tip moment of inertia has a large influence on the frequency despite the fact that this effect decreases, but never vanishes, for large values of contact stiffness. This means that, for all values of the contact stiffness, the tip moment of inertia influences the frequency for the even modes; however, this influence is insignificant for large values of contact stiffness.

4. Conclusion

In this paper, a new model for the flexural vibration of an atomic force microscope cantilever has been proposed. The parameters that have been neglected in previous investigation are considered in this model. We modelled the interaction using a combination of a set of springs and dashpots to investigate the effects of damping on the frequency. First, we analysed the effect of the angle between the cantilever and the sample surface. The analysis showed that, generally, increase in the angle initially increases the resonant frequency, which then decreases to a constant value for large values of the angle. Moreover, the slope of the frequency variation with respect to the contact position.
increases as the angle increases. Second, we found that the greatest influence of damping occurred in instances when the contact stiffness of the system is small. Finally, by investigating the influence of the tip moment of inertia on the frequency, we found that the variations of the odd and even modes are separately the same. For the odd modes, the tip moment of inertia does not have any effect on the resonant frequency for small contact position values. However, gradually increasing the contact stiffness of the system also increases the sensitivity. In contrast, the tip moment of inertia has a large effect on the resonant frequency for small contact position values but has a small effect for the large values.

References